Math 55 midterm exam, March 20, 2024

This exam consisted of five problems, each worth 7 points. The maximum possible score on the exam was thus 35.

A comment on the cover sheet: **It is OK to leave factorials and expressions like $C(n, r)$ or $P(n, r)$ in your answers.**

1. At least 500 of 600 graduating seniors came to a commencement. When these students were divided into groups of 6, 8 and 11, three students were left over in each case. How many students came to the commencement? [For all problems, explain your reasoning in complete English sentences.]

2. Let p be a prime number other than 2, 3 or 5. Show that p divides the sum

$$10^{p-2} + 10^{p-3} + \cdots 10^2 + 10 + 1;$$

for example, 13 divides the 12-digit number 111111111111.
[It might be helpful to use the identity $x^n - 1 = (x-1)(1+x+x^2+\cdots+x^{n-1})$.]

3. Consider 3-digit strings of distinct letters (e.g., `BER`, `KLY`, `STN`, `FRD`, `SFO`). How many such strings contain at least one vowel (`A`, `E`, `I`, `O`, `U`)?

4. The inequality

$$x + y + z + w \leq 55 \qquad (*)$$

states that the difference $55 - (x + y + z + w)$ is nonnegative. Find the number of solutions to $(*)$ with x, y, z and w nonnegative integers.

5. Computing the gcd of 21 and 8 creates these equations:

$$\begin{aligned} 21 &= 2 \cdot 8 + 5, \\ 8 &= 1 \cdot 5 + 3, \\ 5 &= 1 \cdot 3 + 2, \\ 3 &= 1 \cdot 2 + 1, \\ 2 &= 2 \cdot 1 + 0. \end{aligned}$$

Let a and b be positive integers for which the computation of $\gcd(a, b)$ also produces five equations. Explain in detail why b is at least 8.

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