Math 55 midterm exam, February 16, 2024

This exam consisted of five problems, each worth 7 points.

1. If p, q and r are propositions, show that

$$\Big((p \to q) \wedge (q \to r)\Big) \longrightarrow (p \to r)$$

is a tautology.

2. Suppose that a and b are positive integers with $\gcd(a, b) = 1$. Let r be a real number for which both r^a and r^b are rational numbers. Prove that r is rational.

3. Let S be a set and let $f : S \to \mathcal{P}(S)$ be a function from S to the power set of S. Prove that

$$\{\, s \in S \,|\, s \notin f(s) \,\}$$

is not in the image of f.

4. Let a be an integer that is congruent to 1 (mod 3). Show that a is congruent (mod 9) to one of the three numbers 1, 4, 7. Show also that a^3 is congruent to 1 (mod 9).

5. Prove that there is an irrational number between every two distinct rational numbers.

We won the Axe