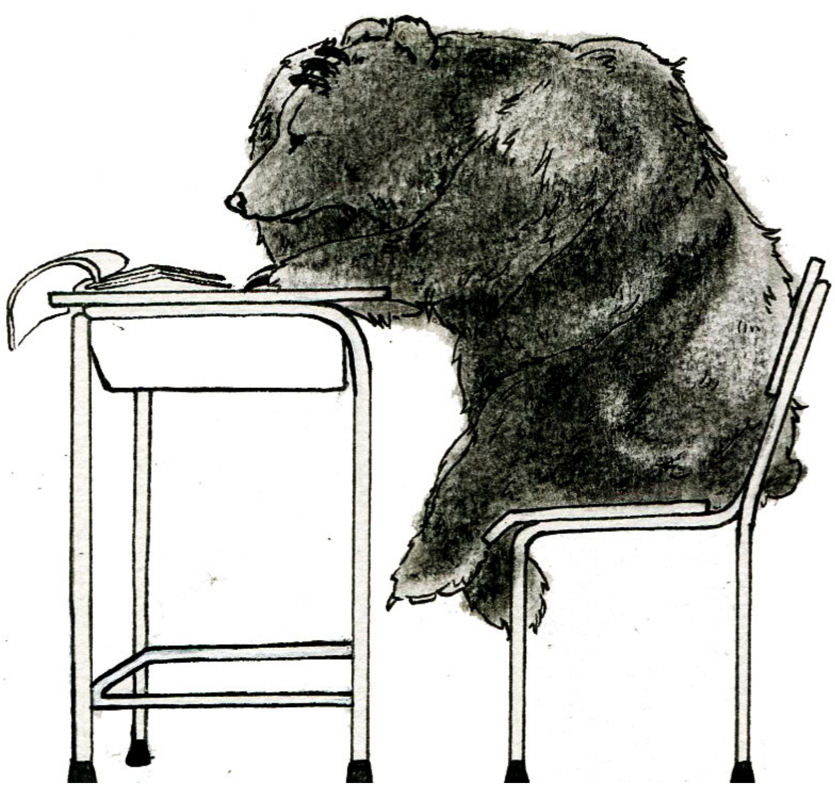
Professor Kenneth A. Ribet
February 18, 2021
8:10–9:30AM

Please do all five problems on this midterm. You have 80 minutes to work on the exam and 15 minutes to upload your work to `Gradescope`. You may consult the textbook, all the material on `bCourses`, the class `piazza` and your own notes. In case of questions, post a private note to instructors on `piazza`. Any clarifications or corrections that need to be promulgated will be added to a pinned post on `piazza`.

Explain all your answers fully; write in complete English sentences.

Not permitted: online searches, other uses of the internet, collaboration with other people (electronic or otherwise). Please act with honesty, integrity and respect for others.



Artwork Ig: `@itchyscabs` `@mimithemimo`

Problem	1	2	3	4	5	Total
Points	4	4	4	4	4	20

1. Using only pencil and paper (and possibly a primitive hand calculator), calculate 7^{888} modulo 4, 25 and 100. Be sure to explain your work and your reasoning. (This applies to all problems.)

2. The number 6 is the product of its proper divisors: $6 = 1 \times 2 \times 3$. So is 8: $8 = 1 \times 2 \times 4$. Which integers > 1 have this property?

3. What is the largest power of 2 dividing the binomial coefficient $\binom{45}{23}$?

4. Since $15^2 - 15 - 1 = 225 - 16 = 209$, 15 is a solution to the congruence

$$x^2 - x - 1 \equiv 0 \pmod{209}.$$

Without doing a major calculation, find another solution to the congruence. Taking note of the factorization $209 = 11 \cdot 19$, determine the number of solutions to the congruence mod 209.

5. Let p be a prime number ≥ 5. This problem is inspired by the first *Challenge* on the slides for our class last Thursday:

a. If there is an element $a \in \mathbf{Z}/p\mathbf{Z}$ satisfying $a^2 + a + 1 = 0$, show that $p \equiv 1$ mod 3.

b. If -3 is a square mod p, show that there is an element $a \in \mathbf{Z}/p\mathbf{Z}$ as in part (a).

To finish: Please copy and sign the statement below.

"*As a member of the UC Berkeley community, I acted with honesty, integrity, and respect for others during this exam. The work that I am uploading is my own work. I did not collaborate with or contact anyone during the exam. I did not seek or obtain solutions from* `chegg.com` *or other sites. I adhered to all instructions for this examination.*"