# Math 110

**PROFESSOR KENNETH A. RIBET**

## Last Examination

## May 14, 2014

11:30AM–2:30PM, 230 Hearst Gym

Your NAME:

Your GSI:

Please put away all books, calculators, cell phones and other devices. You may consult a single two-sided sheet of notes. Please write carefully and clearly in *complete sentences.* (There is lots of time!) Your explanations are your only representative when your work is being graded.

Vector spaces are over $\mathbf{F}$, where $\mathbf{F} = \mathbf{R}$ or $\mathbf{F} = \mathbf{C}$. They may be infinite-dimensional if there is no indication to the contrary. A "real" vector space is a vector space over $\mathbf{R}$.

*At the conclusion of the exam, hand your paper in to your GSI.*

| Problem | Your score | Possible points |
|---|---|---|
| 1 | | 8 points |
| 2 | | 8 points |
| 3 | | 8 points |
| 4 | | 8 points |
| 5 | | 7 points |
| 6 | | 7 points |
| Total: | | 46 points |

*As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.*

**1.** Label each of the following assertions as TRUE or FALSE. Along with your answer provide a proof or counterexample.

**a.** Suppose that $T \in \mathcal{L}(V)$ with $V$ of finite dimension $n$. If $V = \operatorname{range}(T) \oplus \operatorname{null}(T)$, then $\operatorname{null} T = \operatorname{null} T^2 = \operatorname{null} T^3 = \cdots = \operatorname{null} T^n$.

**b.** If $V$ is a finite-dimensional complex vector space, and if $T$ is an operator on $V$, then $T^k$ is diagonalizable for some positive integer $k$.

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**2.** Label each of the following assertions as TRUE or FALSE. Along with your answer provide a proof or counterexample.

**a.** If $V = U \oplus X$ and $V = U \oplus Y$, then $X = Y$.

**b.** If $V$ is a finite-dimensional vector space and $U$ is a subspace of $V$, then every linear functional on $U$ can be extended to a linear funtional on $V$.

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**3.** Label each of the following assertions as TRUE or FALSE. Along with your answer provide a proof or counterexample.

**a.** If an operator on a real finite-dimensional inner product space $V$ has a symmetric matrix with respect to one orthonormal basis of $V$, then it has a symmetric matrix with respect to all orthonormal bases of $V$.

**b.** A normal operator on a complex finite-dimensional inner-product space is self-adjoint if and only if all of its eigenvalues are real.

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**4.** Label each of the following assertions as TRUE or FALSE. Along with your answer provide a proof or counterexample.

**a.** If $v$ is a non-zero vector in a finite-dimensional vector space $V$, there is a basis $(v_1, \ldots, v_n)$ of $V$ such that

$$v = v_1 + v_2 + \cdots + v_n.$$

**b.** If $U$ is a finite-dimensional subspace of an inner-product space $V$ (possibly infinite-dimensional), then $V = U \oplus U^{\perp}$.

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**5.** Suppose that $T$ is an operator on a finite-dimensional real vector space $V$ and that $T$ satisfies $T^2 + 4T + 5I = 0$.

**a.** If $V$ is non-zero, show that there is a $T$-invariant subspace of $V$ whose dimension is 2.

**b.** Show that $\dim V$ is even.

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**6.** Suppose that $V$ and $W$ are finite-dimensional vector spaces and that $T_1$ and $T_2$ are linear maps from $V$ to $W$. If the range of $T_1$ is contained in the range of $T_2$, show that there is an operator $S \in \mathcal{L}(V)$ such that $T_1 = T_2 S$.

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