Math 110

PROFESSOR KENNETH A. RIBET

Final Examination

May 10, 2010

11:30AM–2:30 PM, 10 Evans Hall

Your NAME:

GSI:

Please put away all books, calculators, and other portable electronic devices—anything with an ON/OFF switch. You may refer to a single 2-sided sheet of notes. For numerical questions, *show your work* but do not worry about simplifying answers. For proofs, write your arguments in complete sentences that explain what you are doing. Remember that your paper becomes your only representative after the exam is over.

Please turn in your exam paper to your GSI when your work is complete.

Problem	Your score	Possible points
1ab		6 points
1cd		6 points
2		7 points
3		7 points
4		8 points
5		8 points
6		8 points
Total:		50 points

1. Label each of the following statements as TRUE or FALSE. Along with your answer, provide a clear justification (e.g., a proof or counterexample).

a. Each system of n linear equations in n unknowns has at least one solution.

b. If A is an $n \times n$ complex matrix such that $A^* = -A$, every eigenvalue of A has real part 0.

c. If W and W' are 5-dimensional subspaces of a 9-dimensional vector space V, there is at least one non-zero vector of V that lies in both W and W'.

d. If T is a linear transformation on $V = \mathbf{C}^{25}$, there is a T-invariant subspace of V that has dimension 17.

2. Let T be a linear transformation on an inner-product space. Show that T^*T and T have the same null space.

3. Let $T : V \to W$ be a linear transformation between finite-dimensional vector spaces over a field F. Show that there is a subspace X of V such that the restriction of T to X is 1-1 and has the same range as T.

4. Suppose that T is a self-adjoint operator on a finite-dimensional real vector space V and that $S : V \to V$ is a linear transformation with the following property: every eigenvector of T is also an eigenvector of S. Show that there is a basis of V in which both T and S are diagonal. Conclude that S and T commute.

5. Use mathematical induction and the definition of the determinant to show for all $n \times n$ complex matrices A that the determinant of the complex conjugate of A is the complex conjugate of $\det A$. (The "complex conjugate" of a matrix A is the matrix whose entries are the complex conjugates of the entries of A.)

6. Let W be a subspace of V, where V is a finite-dimensional vector space. Assume that W is a *proper* subspace of V (i.e., that it is not all of V). Show that there is a non-zero element of V^* that is 0 on each element of W. (A harder version of this problem was on the second midterm.)