MATH 54 FINAL

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For each subset of the vector space of 2×2 matrices, determine whether or not it is a vector subspace & justify your answer.

a) $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ s.t. } a+b+c+d=0 \right\}$

b) $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ s.t. } abcd=0 \right\}$

Let $A = \begin{pmatrix} 2 & 2 \\ -2 & 1 \\ 1 & -2 \end{pmatrix}$. For each $\vec{b}$, solve $A\vec{x} = \vec{b}$ if possible; if it is not possible, find the least-squares solution to $A\vec{x} = \vec{b}$ (i.e. find $\vec{x}$ s.t. $\|A\vec{x} - \vec{b}\|$ is as small as possible.)

a) $\vec{b} = \begin{pmatrix} 12 \\ -9 \\ 3 \end{pmatrix}$

b) $\vec{b} = \begin{pmatrix} 12 \\ -9 \\ 5 \end{pmatrix}$

Find an orthogonal matrix P & a diagonal matrix D s.t. $\begin{pmatrix} 13 & 6 \\ 6 & 8 \end{pmatrix} = P^t D P$.

(Reminder: A is orthogonal means $A^t A = I$).

Recall from the homework that $H_0(x)=1$, $H_1(x)=2x$, & $H_2(x)=4x^2-2$ are orthogonal with respect to the inner product $\langle f,g\rangle = \int_{-\infty}^{\infty} e^{-x^2} f(x) g(x)\,dx$.

Compute $$\frac{\int_{-\infty}^{\infty} \left[e^{-x^2} x^2 (4x^2-2)\right] dx}{\int_{-\infty}^{\infty} \left[e^{-x^2} (4x^2-2)^2\right] dx}$$

Find the general solution to the differential equation:

$$y''(x) - y'(x) - 2y(x) = \cos x - \sin 2x$$

Find the Fourier series of

$$f(x) = \begin{cases} 2, & \pi > x \geq 0 \\ -2, & 0 > x > -\pi \end{cases}$$

What value does the Fourier series converge to at $x = 0$?

Find a formal solution to the wave equation with the given initial-boundary conditions.

$$\frac{\partial^2 u}{\partial t^2} = 64 \frac{\partial^2 u}{\partial x^2} \qquad 0 < x < \pi, \; t > 0$$

$$u(0,t) = u(\pi,t) = 0, \quad t > 0$$

$$u(x,0) = 2 \qquad 0 < x < \pi$$

$$\frac{\partial u}{\partial t}(x,0) = \sin 7x - \frac{1}{2}\sin 19x \qquad 0 < x < \pi$$