Prof. Bjorn Poonen
February 25, 2004

MATH 185 MIDTERM 1

Do not write your answers on this sheet. Instead please write your name, your student ID, and all your answers in your blue books. Total: 100 pts., 50 minutes.

(1) (5 pts. each) For each of (a)-(d) below: If the statement is true, write TRUE. If the statement is false, write FALSE. (Please do not use the abbreviations T and F, since in handwriting they are sometimes indistiguishable.) No explanations are required in this problem.

(a) If a function $f(z)$ is holomorphic on all of $\mathbb{C}$, then so is $\overline{f(z)}$.

(b) There exists a branch of $\log(z-2)$ defined on the open unit disk $\{ z \in \mathbb{C} : |z| < 1 \}$.

(c) If $s(z)$ is a branch of $\sqrt{z}$ on an open connected subset of $\mathbb{C}$ containing 4 and $2i$ and if $s(4) = 2$, then $s(2i) = 1 + i$.

(d) There exists a formal power series $f(T)$ such that

$$(4 + T^5) f(T) = \sum_{n=2}^{\infty} n!\, T^n.$$

(2) (10 pts.) Express $\dfrac{(\sqrt{3}+i)^{15}}{(1-i)^{29}}$ in $a + bi$ form.

(3) (10 pts.) Give an explicit formula for a path $f\colon [a, b] \to \mathbb{C}$ (for real numbers a and b of your choice) that traces a circle of radius 2 centered at $7 + 3i$ once around in the *clockwise* direction.

(4) (15 pts.) Find all complex solutions to the equation $\sin z = 1/2$. Show your work.

(5) (10 pts.) For which $z \in \mathbb{C}$ does $\sum_{n=1}^{\infty} e^{2\pi i/n} z^n$ converge? Explain your answer.

(6) (15 pts.) Does the series $\sum_{n \geq 2} \frac{1}{(z+n)^2}$ converge uniformly in the closed unit disk $\{ z : |z| \leq 1 \}$? Explain your answer.

(7) (10 pts. each) Let $S = \{ z \in \mathbb{C} : 0 \leq \mathrm{Re}(z) \leq 1 \text{ and } \mathrm{Im}(z) \geq 0 \}$.

(a) Sketch the image of S under the map $z \mapsto (2+i)z - 3$. Label "corners" with the corresponding complex numbers.

(b) Sketch the image of S under the map $z \mapsto e^{iz}$. Label "corners" with the corresponding complex numbers.

This is the end! At this point, you may want to look over this sheet to make sure you have not omitted any problems. Check that your answers make sense! Please take this sheet with you as you leave.