Final Exam, Friday, 5/14/04
8:10AM – 11:00AM
Math 32, Spring 2004
Instructor: Benjamin Johnson

Student's name: _____

GSI: _____

Do not open your exam until instructed to do so.
Please read all directions carefully.
Please simplify your answers as much as possible.
Please draw a box around all your final answers.
You may not use a calculator on this exam.

The exam consists of 16 questions, plus one bonus question. The point values for each question are indicated below, and also before the problem numbers.

You will have 2 hours and 50 minutes to complete this exam. Please work carefully, and check your answers when you are done. Remember not to spend too much time on any one problem. If you get stuck on a difficult problem, move on to a problem that you know how to do, and come back to the difficult problem later.

If you finish your exam before 10:50AM, you may turn in your exam and leave the room. Solution sheets will be distributed to all those who turn in their exams after 10:00AM. If you leave before 10:00AM, you will not receive a solution sheet. Once you leave the room and collect a solution sheet, you may not return until after the exam is over and all the exams have been collected. Also, you may not leave during the last 10 minutes of the exam.

Please do not write anything below this line.

Problem 1 _____ (out of 6)

Problem 2 _____ (out of 6)

Problem 3 _____ (out of 6)

Problem 4 _____ (out of 6)

Problem 5 _____ (out of 6)

Problem 6 _____ (out of 6)

Problem 7 _____ (out of 6)

Problem 8 _____ (out of 6)

Problem 9 _____ (out of 8)

Problem 10 _____ (out of 6)

Problem 11 _____ (out of 6)

Problem 12 _____ (out of 6)

Problem 13 _____ (out of 8)

Problem 14 _____ (out of 6)

Problem 15 _____ (out of 6)

Problem 16 _____ (out of 6)

Bonus Problem _____ (out of 6)

Total Score _____ (out of 100 + 6)

Problem 1 (6 points)

Solve the following quadratic equation using any method you choose. (Hint: use the quadratic formula.)

$2x^2+3x-7=0$

Problem 2 (6 points)

Sketch a graph of the following equations:

a. $y=|x|$ b. $y=\frac{1}{x}$ c. $y=\sqrt{1-x^2}$

Problem 3 (6 points)

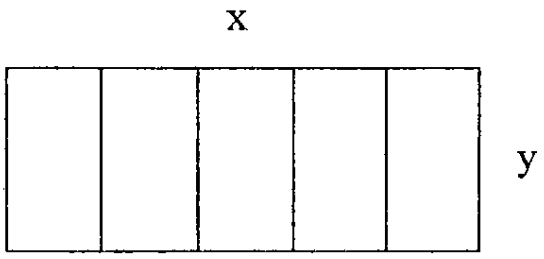
Let $f(x)=4x^2+3$. Let $g(x)=2x-1$. Find an explicit defining formula for the function $f \circ g^{-1}$. Simplify your answer.

Problem 4 (6 points)

Given that f is a linear function, $f(2)=-1$, and $f(5)=-7$, find a defining formula for f.

Problem 5 (6 points)

A farmer wants to build a system of adjacent fenced-in areas to keep his animals separated. His design entails five rectangular pens of equal size, arranged as in the figure below. If he only has enough money for 600 feet of fencing material, what is the maximum total area that he can enclose?



Problem 6 (6 points)

Sketch a graph of the function $g(x)=\frac{x}{x^2-9}$. Be sure to include any important features in your graph including asymptotes, and any x- or y- intercepts.

Problem 7 (6 points)

Sketch a graph of the function $f(x) = -e^{(x-5)}$. Specify or label on your graph any asymptotes or intercepts.

(Hint: first sketch a graph of $y = e^x$ and then use what you know about translation and reflection).

Problem 8 (6 points)

Solve the following inequality. (Hint: Be sure to consider the domain of this original expression when determining your final answer).

$$\log_{10} x + \log_{10}(x-2) \leq \log_{10} 24$$

Problem 9 (8 points)

Complete the following table.

θ	$45°$	$\pi/3$	$-270°$	$11\pi/6$
$\sin\theta$				
$\cos\theta$				
$\tan\theta$				
$\csc\theta$				
$\sec\theta$				
$\cot\theta$				

Problem 10 (6 points)

A wheel with radius 12 inches makes 6 revolutions per second.

a. (3 points) Find the angular speed ω in units of radians/sec.

b. (3 points) Find the linear speed v of a point on the circumference of the wheel in units of inches/sec.

Problem 11 (6 points)

Sketch a graph of the function $h(t)=3\csc \pi t$ for <u>two</u> full periods.

(Hint: Start by graphing $y=\sin \pi t$ for two periods, using what you know about the graph of $y=A\sin(Bx-C)$; then graph $y=\csc \pi t$, and finally $y=3\csc \pi t$).

Problem 12 (6 points)

Prove $\dfrac{\cos(s-t)}{\cos s \sin t} = \cot t + \tan s$.

(Hint: start with the left side, use the subtraction formula for cosine, and then "simplify" to obtain the right side).

Problem 13 (8 points)

Let $z=2+3i$, and let $w=7-i$. Express each of the following in standard rectangular form, (i.e. in the form $a+bi$, where a and b are real numbers).

a. $\bar{z}$

b. $z+w$

c. zw

d. $\frac{z}{w}$

Problem 14 (6 points)

Find all solutions to the equation $x^3-3x^2-9x-5=0$.
(Hint: start by using the rational roots theorem to list the possible rational roots, then find one by guessing and using synthetic division).

Problem 15 (6 points)

Use mathematical induction to prove that for every positive integer n, $\sum_{i=1}^{n} 2i = n(n+1)$. (Hint: $\sum_{i=1}^{n} 2i = 2+4+6+...+2n$).

Problem 16 (6 points)

Compute $\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)^{10}$. Express your answer in standard rectangular form. (Hint: First convert $-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$ to trigonometric form, then use DeMoivre's theorem).

Bonus (6 points)

a. State the Remainder Theorem.

b. If you were to express the polynomial $f(x)=(x^2-9)^{13}+2x-4$ in its standard form, and then use synthetic division to divide the result by $x-3$, what would the remainder be? (Hint: Do not attempt to expand $(x^2-9)^{13}$ or carry out the synthetic division; use part a.).