Math 128A Final 2003 May 21. R. Borcherds

Please make sure that your name is on everything you hand in.
You are allowed calculators and 1 page of notes.
All questions have about the same number of marks.

1. Determine the free cubic spline that approximates the data $f(-1) = 1$, $f(0) = 0$, $f(1) = 1$.

2. Use the modified Euler method

$$w_{i+1} = w_i + (h/2)(f(t_i, w_i) + f(t_{i+1}, w_i + hf(t_i, w_i)))$$

to approximate the solution to $y' = 1 + (t - y)^2$, $2 \le t \le 3$, $y(2) = 1$ with $h = \cdot 5$.

3. Find constants a, b, c so that the formula

$$\int_0^2 f(x)dx = af(0) + bf(1) + cf(2)$$

is exact whenever f is a polynomial of degree at most 2. (You should show how to derive these constants: just quoting them will not get much credit.)

4. Derive the formula $w_{i+1} = w_i + h((3/2)f(t_i, w_i) - (1/2)f(t_{i-1}, w_{i-1}))$ for the Adams-Bashforth two step explicit method by using the Lagrange form of the interpolating polynomial.

5. Estimate $y(1)$ where $y' = -10y$ and $y(0) = 1$ using the forward Euler method $w_{i+1} = w_i + hf(t_i, w_i)$ and the backward Euler method $w_{i+1} = w_i + hf(t_{i+1}, w_{i+1})$ with step size $h = \cdot 5$. Which method is better for this example?

6. For the following linear system

$$x - ay = 1$$
$$ax - y = -1$$

describe for which values of a the system has an infinite number of solutions, no solutions, and exactly one solution, and find the solution when it is unique.

7. Write A in the form LDL^t where A is

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix},$$

L is lower triangular with 1's on the diagonal, and D is diagonal.

8. Find a permutation matrix P, a lower triangular matrix L with 1's on the diagonal, and an upper triangular matrix U so that $PA = LU$ where A is

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & 0 \\ 0 & 1 & -1 \end{pmatrix}.$$

9. Use the Runge-Kutta method of order 4 given by

$$\begin{aligned} k_1 &= hf(t_i, w_i) \\ k_2 &= hf(t_i + h/2, w_i + k_1/2) \\ k_3 &= hf(t_i + h/2, w_i + k_2/2) \\ k_4 &= hf(t_i + h, w_i + k_3) \\ w_{i+1} &= w_i + k_1/6 + k_2/3 + k_3/3 + k_4/6 \end{aligned}$$

with a step size of $h = 1$ to approximate the value of $y(1)$, given that $y' = y$, $y(0) = 1$.

10. Use Taylor's method of order 2 with a step size of $h = \cdot 5$ to estimate $y(1)$, given that $y' = y$, $y(0) = 1$.