P. Vojta

Math 1BM Second Midterm

Thu 23 Mar 2000

1. (12 points) Is the series $\sum_{n=1}^{\infty} \frac{1}{2n^2 - \sqrt{n}}$ absolutely convergent, conditionally convergent, or divergent?

2. (14 points) Describe how one can compute $\sum_{n=1}^{\infty} \frac{1}{n^3}$ to within 0.00005.
(You do not need to actually carry out the computation, but if your answer involves, say, the n^{th} partial sum, then you should say what n is.)

3. (12 points) Is the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ absolutely convergent, conditionally convergent, or divergent?

4. (12 points) Is the series $\sum_{n=1}^{\infty} a_n$, where

$$a_n = \begin{cases} \frac{1}{n+\sqrt{n}}, & \text{if } n \text{ is odd, or} \\ -\frac{1}{n} & \text{if } n \text{ is even} \end{cases}$$

absolutely convergent, conditionally convergent, or divergent? Explain.
[Fewer than 10% of the students even got so far as to approach the main difficulty of this problem.]

5. (18 points) (a). Find the Taylor polynomial, $T_3(x)$, for $f(x) = xe^x$ (centered about $a = 0$).
(b). Use Taylor's Inequality to find an upper bound for the error in using your answer to (a) to compute $f(1)$.

6. (20 points) (a). Show that the series

$$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n+1)}$$

is a solution of the differential equation

$$y' = 1 + xy\ .$$

(b). Over what interval is it a solution?

7. (12 points) Find the curve through the point $(1,1)$ that is everywhere orthogonal to the family of curves $y = Cx^3$.