INFORMATION ON THE PRELIMINARY EXAMINATION

1. General

The Preliminary Examination consists of six hours of written work over a two-day period. It is offered twice a year, typically on the Monday and Tuesday mornings of the week before classes in each Fall and Spring semester, so that results can be announced before classes begin. A student who does not pass the exam during the first three semesters of matriculation in the Ph. D. program will not be permitted to remain in the program. (Under very rare circumstances, exceptions to this rule are made. The authority to grant exceptions rests with the department's Committee Omega.) The policy is three semesters, not three tries: a student who chooses not to take the exam in a semester forfeits that chance to pass. Moreover, students' files record only the date the exam is passed, not failed attempts or numerical scores. Therefore students are encouraged to take the exam every semester until they pass.

2. Purpose of the Preliminary Examination

The exam is designed to test a working knowledge and understanding of the core of an "honors" undergraduate mathematics major program. The emphasis is on the application of theorems and basic principles (rather than on proofs of the standard theorems) and on the ability to construct coherent, sound arguments. The purpose of the examination is to determine whether first-year students in the Ph. D. program have mastered basic mathematics thoroughly enough to continue with a reasonable chance of success. The purpose is *not* to eliminate a predetermined percentage of students from the program. In taking the examination, students are not competing against each other; rather, they are trying to meet the standards set by the Preliminary Examination Committee on behalf of the faculty. Students should expect these standards to be rigorous.

3. Syllabus for the Preliminary Examination

The student taking the examination should be familiar with the material outlined below.

3.1. **Calculus (Mathematics 1A-1B-53-54).** Basic first- and second-year calculus. Derivatives of maps from $\mathbb{R}^m$ to $\mathbb{R}^n$, gradient, chain rule; maxima and minima, Lagrange multipliers; line and surface integrals of scalar and vector functions; Gauss's, Green's and Stokes' theorems. Ordinary differential equations; explicit solutions of simple equations.

Date: December 9, 2003.

3.2. **Classical analysis (Mathematics 104).** Topology of $\mathbb{R}^n$ and metric spaces; properties of continuous functions, compactness, connectedness, limit points; least upper bound property of $\mathbb{R}$. Sequences and series, Cauchy sequences, uniform convergence and its relation to derivatives and integrals; power series, radius of convergence, Weierstrass M-test; convergence of improper integrals. Compactness in functions spaces. Inverse and implicit function theorems and applications; the derivative as a linear map; existence and uniqueness theorems for solutions of ordinary differential equations; elementary Fourier series. Texts: Rosenlicht, *Introduction to analysis*, Dover; Marsden, *Elementary classical analysis*, Freeman; Bartle, *Elements of real analysis*, Wiley; Rudin, *Principles of mathematical analysis*, McGraw-Hill.

3.3. **Abstract algebra (Mathematics 113).** Elementary set theory, e. g. uncountability of $\mathbb{R}$. Groups, subgroups, normal subgroups, homomorphisms, quotient groups, automorphisms, groups acting on sets, Sylow theorems and applications, finitely generated abelian groups. Examples: permutation groups, cyclic groups, dihedral groups, matrix groups. Basic properties of rings, units, ideals, homomorphisms, quotient rings, prime and maximal ideals, fields of fractions, Euclidean domains, principal ideal domains and unique factorization domains, polynomial rings. Elementary properties of finite field extensions and roots of polynomials, finite fields. Texts: Lang, *Algebra*, Addison-Wesley; Hungerford, *Algebra*, Springer; Herstein, *Topics in algebra*, Wiley.

3.4. **Linear algebra (Mathematics 110).** Matrices, linear transformations, change of basis; nullity-rank theorem. Eigenvalues and eigenvectors; determinants, characteristic and minimal polynomials, Cayley-Hamilton theorem; diagonalization and triangularization of operators; Jordan normal form, rational canonical form; invariant subspaces and canonical forms; inner product spaces, hermitian and unitary operators, adjoints. Quadratic forms. Texts: Noble-Daniel, *Applied linear algebra*, Prentice-Hall; Hoffman-Kunze, *Linear algebra*, Prentice-Hall; Strang, *Linear algebra*, Academic Press.

3.5. **Complex analysis (Mathematics 185).** Basic properties of the complex number system. Analytic functions, conformality, Cauchy-Riemann equations, elementary functions and their basic properties (rational functions, exponential function, logarithm function, trigonometric functions, roots, e. g. $\sqrt{z}$). Cauchy's theorem and Cauchy's integral formula, power series and Laurent series, isolation of zeros, classification of isolated singularities (including singularity at ∞), analyticity of limit functions. Maximum principle, Schwarz's lemma, Liouville's theorem, Morera's theorem, argument principle, Rouché's theorem. Basic properties of harmonic functions in the plane, connection with analytic functions, harmonic conjugates, mean value property, maximum principle. Residue theorem, evaluation of definite integrals. Mapping properties of linear fractional transformations, conformal equivalences of the unit disk with itself and with the upper half-plane. Texts: Marsden, *Basic complex analysis*, Freeman; Ahlfors, *Complex analysis*, McGraw-Hill; Conway, *Functions of one complex variable*, Springer.

4. Grading policy

Each participant is assigned a number, and the identity of persons who pass or fail is not known until after decisions have been made. Each problem is graded by a single committee member, on a scale ranging from 0 to 10. The policy is not to give credit for inconclusive argumentations, so as not to penalize students who are aware of their inability to solve a problem and accordingly write nothing down. Partial credit is awarded only when substantial progress has been made toward a solution. After the examination has been graded, the committee meets to decide on passes and failures. The committee first selects two scores, a high one corresponding to a performance so good that it clearly deserves a passing grade, and a low one corresponding to a performance so poor that it clearly deserves a failing grade. Papers with scores between those two thresholds are examined individually and discussed by the committee until a consensus is reached on the minimum passing score. That score varies from examination to examination, because, naturally, the examinations are not of uniform difficulty. The committee must exercise its collective judgment in deciding what constitutes a passing performance on the examination. As mentioned in section 2, there is no predetermined failure rate.

Students will be given a chance to inspect their graded exams in 910 Evans. They should in particular check that no mistakes have been made in transferring and adding the scores. Students should not remove their exams from 910 Evans or make marks on them. Students with questions should submit them in writing by filing a *Preliminary Examination review request form*, obtainable in 910 Evans. It is requested that a review request form be filed only if it could change the outcome of the exam or if there is a substantial mathematical point involved.