Math 1B: Final Exam
Friday, 14 August 2009

Instructor: Theo Johnson-Freyd
http://math.berkeley.edu/~theojf/09Summer1B/

Name: ____________________________________

Problem Number	1	2	3	4	5	6	7	Total
Score								
Maximum	20	20	10	15	10	10	15	100

Please do not begin this test until 2:10 p.m. You may work on the exam until 4 p.m. Please do not leave during the last 15 minutes of the exam time.

You must always justify your answers: show your work, show it neatly, and when in doubt, use words (and pictures!) to explain your reasoning. Please box your final answers.

Calculators are not allowed. Please sign the following honor code:

I, the student whose name and signature appear on this midterm, have completed the exam by myself, without any help during the exam from other people, or from sources other than my allowed one-page hand-written cheat sheet. Moreover, I have not provided any aid to other students in the class during the exam. I understand that cheating prevents me from learning and hurts other students by creating an atmosphere of distrust. I consider myself to be an honorable person, and I have not cheated on this exam in any way. I promise to take an active part in seeing to it that others also do not cheat.

Signature: ____________________________________

1. (20 pts total – 2 pts each) For each of the following statements, determine if the conclusion ALWAYS follows from the assumptions, if the conclusion is SOMETIMES true given the assumptions, or if the conclusion is NEVER true given the assumptions. You do not need to show any work or justify your answers for these question: only your answer will be graded.

   (a) (2 pts) If $\lim_{n\to\infty} a_n < 1$, then $\sum_1^\infty a_n$ converges.

   ALWAYS SOMETIMES NEVER

   (b) (2 pts) If the series $\sum_1^\infty a_n$ and $\sum_1^\infty b_n$ both converge, then $\sum_1^\infty (a_n + b_n)$ converges.

   ALWAYS SOMETIMES NEVER

   (c) (2 pts) If the sequences $\{a_n\}$ and $\{b_n\}$ both diverge, then $\{a_n b_n\}$ diverges.

   ALWAYS SOMETIMES NEVER

   (d) (2 pts) If $\sum_1^N a_n \geq -10$ for every N and if $a_n \leq 0$ for every n, then $\sum_1^\infty a_n$ converges.

   ALWAYS SOMETIMES NEVER

   (e) (2 pts) If $\sum_1^\infty c_n 2^n$ converges absolutely, then $\sum_1^\infty c_n(-2)^n$ converges conditionally.

   ALWAYS SOMETIMES NEVER

(f) (2 pts) If $\sum_1^\infty c_n(-2)^n$ converges conditionally, then $\sum_1^\infty c_n$ converges absolutely.

ALWAYS SOMETIMES NEVER

(g) (2 pts) If $\{b_n\}$ is a decreasing positive sequence, then $\sum_1^\infty (-1)^n b_n$ converges.

ALWAYS SOMETIMES NEVER

(h) (2 pts) If p is a real number, then the Ratio Test can be used to determine whether $\sum_1^\infty 1/n^p$ converges.

ALWAYS SOMETIMES NEVER

(i) (2 pts) If $0 \le a_n \le b_n$ and $\sum_1^\infty a_n$ converges, then $\sum_1^\infty b_n$ converges.

ALWAYS SOMETIMES NEVER

(j) (2 pts) If $\lim_{n\to\infty} b_n$ exists, then $\sum_1^\infty (b_n - b_{n+1})$ converges to b_1.

ALWAYS SOMETIMES NEVER

2. (20 pts total – 5 pts each) Determine whether each of the following series is ABSOLUTELY CONVERGENT, CONDITIONALLY CONVERGENT, or DIVERGENT. You must specify which test(s) you use for each series, and why the series satisfies the conditions of the test.

   (a) (5 pts) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n+1)}{n!}$

   (b) (5 pts) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{n+1}$

(c) (5 pts) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(d) (5 pts) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3 + \sqrt{n}}$

3. (10 pts) Find the interval of convergence of the following power series:

$$\sum_{n=2}^{\infty} \frac{(x-4)^n}{n \ln n \, 2^n}$$

4. (15 pts) Evaluate the following definite integral as a series.

$$\int_0^1 \sqrt[3]{1+x^6}\,dx$$

You must both:

- Write your final answer in Σ notation, but you may leave your answer in terms of the binomial coefficients $\binom{k}{n}$.
- Write out the first four terms of the series.

5. (a) (5 pts) Find power series representations (centered at 0) for each of the following functions, and state the intervals of convergence for each series

- $\ln(1-x)$

- $\ln(1-x^2)$

- $\ln(1+x)$

(b) (5 pts) Show that as power series, the representations for the above functions satisfy

$$\ln(1-x^2) = \ln(1-x) + \ln(1+x)$$

6. (a) (2 pts) State the power series representation for $\arctan(x)$ centered at 0. What is its interval of convergence?

(b) (4 pts) What is $\tan \pi/6$? Use this value for x in the power series representation to find a series that converges to $\pi/6$. Is the convergence absolute or conditional?

(c) (4 pts) Use the error estimate for an Alternating Series to determine how many summands you would need in order to use this series to estimate $\pi/6$ correct to three decimal places ($|\text{error}| < 0.001$).

7. (15 pts) Find a power-series representation (centered at 0) for the solution to the following initial value problem:
$$y'' - xy' - 2y = 0, \;\; y(0) = 1, \;\; y'(0) = 1$$

References: All the problems on this midterm are due to the instructor, although they are loosely based on the material in *Single Variable Calculus: Early Transcendentals for UC Berkeley* by James Stewart. The honor-code language is adapted from the Stanford Honor Code (`http://www.stanford.edu/dept/vpsa/judicialaffairs/guiding/honorcode.htm`) and from the exams by Zvezda Stankova.

Feel free to use this page for extra scrap work.