Mathematics 1BM — Professor K. A. Ribet

Final Exam — May 17, 1997

10 Evans

8–11 AM

Your Name: ______________________________ TA: ________________________

This booklet comprises a cover sheet, eight pages of questions, and a formula sheet at the back. Please check that your booklet is complete and write your name on this cover sheet and the question sheets. Some of the questions have low point values; leave them until the end if you can't do them right away. As usual:

- You need not simplify your answers unless you are specifically asked to do so.
- It is essential to write legibly and *show your work*.
- If your work is absent or illegible, and your answer is not perfectly correct, then no partial credit can be awarded.
- Completely correct answers which are given without justification may receive little or no credit.
- During this exam, you are not allowed to use calculators or consult your notes or books.

Problem	Maximum	Your Score
1	8	
2	8	
3	9	
4	7	
5	19	
6	8	
7	9	
Total	**68**	

At the conclusion of the exam, hand in this exam paper to your TA.

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1a *(3 points)*. Express as a definite integral the length of the curve $y = \sin x$, $0 \le x \le \pi/2$.

1b *(5 points)*. Decide whether $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{2^n n!}{5\cdot 8\cdot 11\cdots(3n+2)}$ converges absolutely, converges conditionally, or diverges.

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2a *(4 points)***.** Find $f^{(666)}(0)$ if $f(x) = \sec\left(x^{333}\right)$.

2b *(4 points)***.** A tank contains 500 L of pure water. Brine that contains 0.05 kg of salt per liter flows into the tank at the rate of 8 L/min. The solution is kept thoroughly mixed and drains from the tank at the rate of 8 L/min. Let $y(t)$ be the amount of salt in the solution at time t, measured in kg; time is measured in minutes. What differential equation is satisfied by y? Without solving this equation, guess the value of $\lim_{t\to\infty} y(t)$.

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3a *(4 points)*. Find one solution to $y'' + 5y' + 6y = \cos x$.

3b *(5 points)*. Use power series methods to solve the initial-value problem:
$y'' + xy' - 2y = 0$, $y(0) = 1$, $y'(0) = 0$.

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4a *(3 points)*. Let $y = f(x)$ be the solution to the initial-value problem $y' = y^2 + 2y$, $y(2) = 1$. Without finding a formula for $f(x)$, compute $f'(2)$ and $f''(2)$.

4b *(4 points)*. Find the equation of the curve which passes through $(1, 1)$ and which is orthogonal to the family of curves $2x^2 + y^2 = k$.

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Evaluate each of the following integrals whenever possible (beware of divergent improper integrals):

5a *(5 points)***.** $\displaystyle\int_0^2 \sqrt{2x - x^2}\, dx.$

5b *(5 points)***.** $\displaystyle\int_{-5}^{5} \frac{4\, dt}{t^2 - 2t - 3}.$

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5c *(5 points)***.** $\displaystyle\int \cos\sqrt{x}\,dx.$

5d *(4 points)***.** $\displaystyle\int_e^\infty \frac{dx}{x(\ln x)^2}.$

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6a *(4 points)*. Test for convergence: $\displaystyle\sum_{n=0}^{\infty} \sqrt{\frac{2^n + 3^n}{6^n}}$.

6b *(4 points)*. How many terms of the alternating series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}$ are required to approximate the actual sum with an error of less than 5×10^{-4}?

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7a *(4 points)***.** Using a series representation for $(1+x)^{1/2}$, evaluate
$$\sum_{n=2}^{\infty}(-1)^n \frac{\big(1\cdot 3\cdot 5\cdots(2n-3)\big)\,9^n}{n!(32)^n}.$$

7b *(5 points)***.** Solve the differential equation $xy' = 2\sqrt{xy} - y$.