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10 Evans Hall

Spring 1996, Math 54, Lecture 2
Second Midterm

19 March, 1996
9:40-11:00 AM

1. (26 points) Let $A = \begin{pmatrix} -1 & -1 & 2 \\ 0 & -1 & 2 \\ 4 & 1 & -2 \end{pmatrix}$.

(a) (8 points) Find the eigenvalues of A.

(b) (7 points) Find a basis of R^3 consisting of eigenvectors of A.

(c) (6 points) Find an invertible matrix P such that $P^{-1}AP$ is diagonal.

(d) (5 points) For P as above, find $P^{-1}AP$. (If you can get the answer without further calculation, do so!)

2. (24 points) Give an *example* of each of the following. You do not have to justify your answers or show any computations.

(a) (4 points) Two 2×2 matrices A and B such that $AB \neq BA$.

(b) (4 points) A nonzero matrix A and a nonzero vector $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{0}$.

(c) (4 points) An inner product $< , >$ on R^2 different from the standard inner product. Describe this by showing explicitly how to evaluate $< \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} >$.

(d) (4 points) A 2×2 orthogonal matrix.

(e) (4 points) A vector space V and a set S which spans V but is not a basis of V.

(f) (4 points) A vector $\mathbf{v}$ which lies in the subspace of R^2 spanned by $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, and such that $\|\mathbf{v}\| = 1$.

3. (15 points) Suppose A is an $m \times n$ matrix, and $\mathbf{b}$, $\mathbf{c}$ are column vectors of height m such that the equations $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{c}$ are both consistent. Show that the equation $A\mathbf{x} = \mathbf{b}+\mathbf{c}$ is also consistent.

4. (20 points) Find a 'least squares solution' to the system of equations

$$\begin{aligned} x_1 + 2x_2 &= 0, \\ x_1 + x_2 &= 2, \\ x_1 &= -2. \end{aligned}$$

5. (15 points) Find all cube roots of $-i$; i.e., all complex numbers $a+bi$ such that $(a+bi)^3 = -i$. (You will get *nearly* full credit if you correctly express these roots in terms of trigonometric functions; full credit if you use the exact algebraic values of those functions.)