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Mathematics H185
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MIDTERM EXAMINATION 2

Open book, open notes. In your proofs you may use any results from Chapters I–VIII of the textbook, including results from the assigned exercises in those chapters. Be clear about what results you use.

The points for each problem are in parentheses.

1. (15) Let a and b be distinct points of $\mathbb{C}$. Find the residues of the function $f(z) = (z-a)^{-1}(z-b)^{-3}$ at a and b.

2. (10) Suppose φ is a linear-fractional transformation that maps the unit disk $\mathbb{D}$ onto itself, and f is a holomorphic self-map of $\mathbb{D}$ satisfying $f(\varphi^{-1}(0)) = 0$. Prove $|f(z)| \leq |\varphi(z)|$ in $\mathbb{D}$.

3. (20) Let γ be the boundary of the square with vertices $0, 1, 1+i, i$, oriented counterclockwise. Evaluate the integrals

$$I_1 = \int_\gamma z^2 dz, \qquad I_2 = \int_\gamma |z|^2 dz, \qquad I_3 = \int_\gamma \mathrm{Re}(z^2) dz.$$

4. (20) (a) Convert the integral

$$I = \int_0^{2\pi} \sin^2\theta \cos^4\theta \; d\theta$$

into a complex integral over the unit circle in the complex plane.

(b) Evaluate the integral found in part (a).