MATH 114

PROFESSOR KENNETH A. RIBET

Last Midterm Examination

April 8, 2004

12:40–2:00 PM

Name:

SID:

Please put away all books, calculators, electronic games, cell phones, pagers, .mp3 players, PDAs, and other electronic devices. You may refer to a single 2-sided sheet of notes. Please write your name on each sheet of paper that you turn in; don't trust staples to keep your papers together. Explain your answers in full English sentences as is customary and appropriate. Your paper is your ambassador when it is graded.

Problem:	Your score:	Total points
1		7 points
2		8 points
3		8 points
4		7 points
Total:		30 points

1. Suppose that K is a subfield of the complex field $\mathbb{C}$ and that $\alpha \in \mathbb{C}$ is algebraic over K. Let E be a field intermediate between K and $K(\alpha)$: $K \subseteq E \subseteq K(\alpha)$. Let

$$p(t) = t^d + a_{d-1}t^{d-1} + \cdots + a_1 t + a_0$$

be the minimal polynomial of α over E. Show that $E = K(a_0, a_1, \ldots, a_{d-1})$.

2. Let $\alpha = \sqrt{3+\sqrt{5}} \approx 2.2882$, and let $K = \mathbb{Q}(\alpha)$. Let L be the splitting field of the minimal polynomial of α.

(a) Find the Galois group $G = \mathrm{Gal}(L : \mathbb{Q})$ of the extension $L : \mathbb{Q}$.

(b) Find all subgroups of G.

(c) For each subgroup H of G, identify the fixed field of H.

3. Let $L = \mathbb{Q}(\sqrt{-3}, \sqrt[3]{2})$ be the splitting field of $t^3 - 2$. How many different fields K (other than $\mathbb{Q}$ and L) satisfy $\mathbb{Q} \subset K \subset L$? For each field K, indicate the degree $[K : \mathbb{Q}]$ and write K in the form $\mathbb{Q}(\alpha)$.

4. Suppose that $p(t)$ is a monic polynomial over $\mathbb{Q}$ and let $p'(t)$ be the derivative of $p(t)$. Suppose that 1 is the highest common factor of $p(t)$ and $p'(t)$ in the ring $\mathbb{Q}[t]$. If n is the degree of p, prove that $p(t)$ has n *distinct* roots in $\mathbb{C}$.