Final

Math 74

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sid :

Please answer the following questions as completely as you can. Justify your steps. Problems will be graded on both completeness and clarity.

Read all of the questions before beginning the exam. The questions are in no order (*e.g.*, you should not assume that the first question is easier than the second).

Problem	Points
1	10
2	10
3	10
4	10
5	10
6	10
7	10
8	10
9	10
10	10
11	10
EC	10
	110

Course Grade

1. Let X be a non-empty set. Is $(\mathcal{P}(X), \cup)$ a group? If so give the data that makes up this group and check the conditions. If not explain why it is not a group.

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2. Prove that $\neg(\exists x \in U : P(x))$ is equivalent to $\forall x \in U, \neg P(x)$.

3. Let $f : A \longrightarrow B$ and $g : B \longrightarrow C$ be surjections. Show that $g \circ f : A \longrightarrow C$ is also a surjection. *You do not need to prove that $g \circ f$ is a function*

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4. Let $S \neq \emptyset$ be a subset of the natural numbers greater than 0. Suppose further that for all $a, b \in S$ either $a|b$ or $b|a$.

Show that there is a $c \in S$ such that $\forall a \in S \quad c|a$.

Be sure to verify all your steps

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5. Define a function on $\mathbb{N} \setminus \{0\}$ as follows:

$f(1) = 3,\ f(2) = 9$ and $\forall n \in \mathbb{N}, n \geq 4 \quad \Rightarrow \quad f(n) = 4f(n-1) - 3f(n-2)$.

Show that $f(n) = 3^n$ for all $n \in \mathbb{N} \setminus \{0\}$.

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6. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be an increasing * function . Show that f is one-to-one.
   * Recall that f is increasing means $x > y \Rightarrow f(x) > f(y)$ for all $x, y \in \mathbb{R}$

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7. Let $A = \{1, 2, 4, 7, 14, 28\}$ and consider A as a poset under the relation xRy iff $x|y$
You do not need to prove that R is an ordering relation.

   (a) Draw the Hasse diagram for (A, R).

   (b) What is the least upper bound of $\{2, 7\}$ in this setting?. *Prove your claim.*

   (c) Consider $\mathbb{Z}_+$ under the relation R (defined above). What is the greatest lower bound of $\{a, b\}$?
   Prove your claim.

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8. Let a and b be integers. Prove that ab is odd iff a and b are odd.

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9. Justify the following claim:

$\bigcup_{\alpha\in\Delta} A_\alpha$ is the smallest set X such that $A_\alpha \subseteq X$ for all $\alpha \in \Delta$.

You should explain what is meant by smallest, and then prove the claim.

10. Let R and S be relations from A to B. Prove or disprove the following:

$$\text{Dom}(R \cap S) = \text{Dom}(R) \cap \text{Dom}(S)$$

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11. Let A be the set of all functions from $\mathbb{R}$ to $\mathbb{R}$. Define a relation R on A as follows: Say that fRg iff $\exists \epsilon \in \mathbb{R} \; : \epsilon > 0$ and $f|_{(-\epsilon,\epsilon)} = g|_{(-\epsilon,\epsilon)}$

(a) Show that R is an equivalence relation.

(b) Describe the partition of A associated to R.

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Extra Credit

One may restrict the equivalence relation defined in Problem 11 to differentiable functions and then consider the relation as making a statement about the derivative at 0 of these functions. What is this statement? In this light give another description of the partition associated to the equivalence relation.

Some pictures may help you here.

Facts you may use

1. You may use (without proof) the familiar facts about the natural ordering on the number systems, *e.g.*, $(\mathbb{R}, \leq)$ is a total ordering.

2. You may use any of the equivalent forms of induction wihtout showing that form of induction is equivalent to weak induction.

3. $\emptyset \cap A = \emptyset$ and $\emptyset \cup A = A$ for any set A.

4. $\forall a, b, \in \mathbb{N} \ \ a|b \ \ \Rightarrow a \leq b$