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Discrete Exterior Calculus with Convergence to the Smooth Continuum

ABSTRACT

A major task of mathematics today is to harmonize the continuous and the discrete, to include them in one comprehensive mathematics, and to eliminate obscurity from both.

E.T. Bell, ``Men of Mathematics'' 1937

In *Analysis Situs*, Poincare started with a simplicial complex as his basic discrete model. This has had broad appeal since a finite set of points determines the simplexes and the boundary operator has a natural, discrete definition. In over a hundred years, this approach has not yielded a full discrete theory and convergence to the smooth continuum has remained elusive.

In this lecture, entirely new definitions of discrete chains are introduced in a Riemannian manifold as idealized, geometric analogues to differential forms. Reminiscent of the shift from simplicial to singular homology, an intrinsic theory of discrete calculus arises which converges to the smooth continuum. The theory lays new foundations for much of standard analysis in a way that captures infinite limiting processes--- the heart of calculus--- in terms of finite computations. Certain constructions of topology are restored to geometry at the level of chains and cochains, without passing to homology and cohomology. Examples include Poincare duality, intersection of chains, and linking number. Smooth manifolds, metrics, fractals, vector fields, curvature, differential forms, foliations and measures can be discretized. Operators on forms have geometric counterparts for chains including Lie derivative, Hodge star, codifferential and Laplace, giving them physical meaning alongside the venerable boundary operator. Newly defined geometric products have natural definitions such as interior, exterior, wedge, cap, cup, slant, and convolution, with interesting commutator relations.

But what are these fluxions? The velocities of evanescent increments? And what are these same evanescent increments? They are neither finite quantities,

nor quantities infinitely small, nor yet nothing. May we not call them the ghosts of departed quantities?

Bishop Berkeley, ``The Analyst'' 1734