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Growth of the Number of Periodic Points of Generic Maps

ABSTRACT

Let M be a compact manifold, and f a C^r-smooth map from M to M. Define the number of isolated periodic points of period n by $$ P_n(f) = \#\{\textrm{ isolated points } x \textrm{ in }M \textrm{ such that } f^n(x)=x\}. $$ Artin and Mazur (1965), using Nash's approximations, proved that for a dense set of C^r-smooth maps, the number of periodic points $P_n(f) $ grows at most exponentially fast, i.e. for some $C > 0$, and all positive integers n, $$ P_n(f) \leq \exp(C n). \leqno(*) $$ For more than thirty years there was no other proof known. During the talk we shall give an elementary proof of this result due to the author. Various questions about the relation of property (*) to the dynamical zeta-function (Smale 1967) and topological entropy (Bowen 1978), and to other properties for generic diffeomorphisms were raised. It turns out that Artin-Mazur property (*) is not generic, which was shown by the author using a theorem of Gonchenko, Shilnikov, and Turaev. Therefore, all the above questions have negative answers. Other aspects of the problem will be discussed.