UC Berkeley Math 10B, Spring 2014: Midterm 2

Prof. Persson, April 15, 2014

Name: _____

SID: _____

Neighbors: Please write the names of the students next to you (or "None"):

Left: _____

Right: _____

Section: Circle your discussion section below:

Time	Sec	Room	GSI
MWF 8-9am	102	4 Evans	Jason Ferguson
MWF 9-10am	103	41 Evans	Jason Ferguson
MWF 10-11am	104	39 Evans	Anna Lieb
MWF 11-12pm	105	39 Evans	Anna Lieb
MWF 12-1pm	106	41 Evans	Zvi Rosen
MWF 1-2pm	101	45 Evans	Zvi Rosen
MWF 2-3pm	107	3113 Etcheverry	Ralph Morrison
MWF 3-4pm	108	103 Moffitt	Ralph Morrison

Other/none, explain: _____

Grading

1	/ 4
2	/ 4
3	/ 6
4	/ 6
5	/ 6
6	/ 6
7	/ 6
8	/ 6
	/44

Instructions:

- Closed book: No notes, no books, no calculators.

- Exam time 80 minutes, do all of the problems.

- You must justify your answers for full credit.

- Write your answers in the space below each problem.

- If you need more space, use reverse side or scratch pages. Indicate clearly where to find your answers.

- You do not have to simplify your answers for any question. More specifically, we will accept answers in terms of any combination of [finite] sums, differences, products, quotients, polynomials, exponents, logs, absolute values, trig functions, inverse trig functions, factorials, $P(n, k)$, $C(n, k)$, $S(n, k)$, and $p_k(n)$.

1. (4 points) You roll a die (possibly biased) 100 times, and 20 of the rolls land on 6. Using this, construct a 95% confidence interval for the probability p for rolling a 6.

2. (4 points) X is a random variable that gives

$$-4, \ 0, \ \text{and} \ 4$$

as outputs with probabilities

$$\frac{1}{2}, \ \frac{1}{4}, \ \text{and} \ \frac{1}{4},$$

respectively, and can give no other outputs. Find $E[X]$ and $\text{Var}[X]$.

3. You independently toss three coins and count the number of heads, and repeat this experiment for a total of 120 times. You get 0 heads 12 times, 1 head 38 times, 2 heads 50 times, and 3 heads 20 times. Suppose you want to test the null hypothesis H_0 that all three coins are fair.

a) (3 points) Construct a table showing the observed frequencies and the expected frequencies under the null hypothesis H_0.

b) (3 points) What is the χ^2-statistic for this data? Describe how to use this to determine if the null hypothesis can be rejected.

4. (6 points) Solve the recursion equation

$$a_0 = 3, \quad a_1 = 0,$$

$$a_n = pa_{n-1} + 2p^2 a_{n-2}, \qquad n = 2, 3, 4, \ldots,$$

where p is a nonzero constant. Leave your answer in terms of p.

5. (6 points) Evaluate the integral $\displaystyle\int \frac{x^2 + 2x - 2}{x^3 + x^2}\, dx.$

5

6. (6 points) Solve the initial value problem $ty'' + 2y' = 12t^2$, $y(1) = 1$, $y'(1) = 1$.
Hint: Let $z = y'$. Solve for z, and then use z to solve for y.

7. (6 points) Find all solutions, if any, to the differential equation $e^{-t}y' = (y+1)^2$.

8. a) (3 points) Show that $y_1(t) = t$ and $y_2(t) = e^{2t}$ are both solutions to the differential equation $(2t - 1)y'' - 4ty' + 4y = 0$.

b) (3 points) Solve the initial value problem

$$(2t - 1)y'' - 4ty' + 4y = 0, \quad y(0) = 1 \text{ and } y'(0) = -3.$$