UC Berkeley Math 10B, Spring 2014: Midterm 1

Prof. Persson, March 4, 2014

Name: ______________________________

SID: ______________________________

Neighbors: Please write the names of the students next to you (or "None"):

Left: ______________________________

Right: ______________________________

Section: Circle your discussion section below:

Sec	Time	Room	GSI
101	MWF 1-2pm	45 Evans	Zvi Rosen
102	MWF 8-9am	4 Evans	Jason Ferguson
103	MWF 9-10am	41 Evans	Jason Ferguson
104	MWF 10-11am	39 Evans	Anna Lieb
105	MWF 11-12pm	39 Evans	Anna Lieb
106	MWF 12-1pm	41 Evans	Zvi Rosen
107	MWF 2-3pm	3113 Etcheverry	Ralph Morrison
108	MWF 3-4pm	103 Moffitt	Ralph Morrison

Other/none, explain: ______________________________

Grading

1	/ 9
2	/ 5
3	/ 5
4	/ 5
5	/ 5
6	/ 6
7	/ 9
	/44

Instructions:

- Closed book: No notes, no books, no calculators.
- Exam time 80 minutes, do all of the problems.
- You must justify your answers for full credit.
- Write your answers in the space below each problem.
- If you need more space, use reverse side or scratch pages. Indicate clearly where to find your answers.
- Unless we ask for an actual number, we will accept answers in terms of any combination of [finite] sums, differences, products, quotients, exponents, factorials, $P(n, k)$, $C(n, k)$, $S(n, k)$, and $p_k(n)$.

1. A license plate consists of four letters (A-Z) followed by three digits (0-9), for example ABCA212. How many different license plates

a) (3 points) are there in total?

b) (3 points) have no repeated letters or digits?

c) (3 points) have exactly two Q's and at least one 9?

2. (5 points) You roll a fair, standard six-sided die [with sides numbered 1 through 6] twice, independently. What is the probability that the number on the second roll is strictly greater than the number on the first roll?

3. (5 points) Consider the set A of the five weekdays,

$$A = \{\text{Mon}, \text{Tue}, \text{Wed}, \text{Thu}, \text{Fri}\}.$$

Compute the number of ways to partition A into at most 3 non-empty subsets, where the order of the subsets does not matter. ("Compute" means give the actual number.)

4. (5 points) Let $r \geq 1$, $n \geq 0$, and $0 \leq k \leq r$ be integers. Find an expression [in terms of r, n, and/or k] for the number of non-negative integer solutions to the equation

$$x_1 + x_2 + \cdots + x_r = n,$$

for which exactly k of the variables x_i are equal to zero.
Hint: First consider the number of ways to choose the k variables that are zero, then find the number of positive integer solutions for the other variables.

5. (5 points) Two cards are randomly chosen without replacement from an ordinary deck of 52 cards. Find the probability that both cards are aces, given that one of the two cards is the ace of spades. ("Ordinary deck" means there are 4 cards out of the 52 that are aces. One of those 4 aces is the ace of spades.)

6. (6 points) Urn A has 5 white and 7 black balls. Urn B has 3 white and 12 black balls. We flip a fair coin. If the outcome is heads, then a ball from urn A is selected at random, whereas if the outcome is tails, then a ball from urn B is selected at random. Suppose that a white ball is selected. What is the probability that the coin landed tails?

7. Suppose an unfair six-sided die is 3 times as likely to land on 6 than each of $1, 2, 3, 4, 5$, which are equally likely.

a) (3 points) Let X denote the value of the die. Find the probability mass function f_X.

b) (3 points) Suppose you roll the die independently 10 times. What is the probability that it lands on 4 or higher exactly 7 times?

c) (3 points) Suppose you roll the die independently until you get a 1. What is the probability that you will need to roll at most 4 times?

(Scratch Paper, Page 1)

(Scratch Paper, Page 2)