

UC Berkeley Math 10A, Fall 2014: Midterm 2

Prof. Persson, November 12, 2014

Name: _____

SID: _____

Section: Circle your discussion section below:

Sec	Time	Room	GSI	Grading	
101	TuTh 8-930am	35 Evans	Noble Macfarlane		
102	TuTh 8-930am	31 Evans	Kevin Donoghue	1	/ 6
103	TuTh 11-1230pm	45 Evans	Noble Macfarlane		
104	TuTh 11-1230pm	41 Evans	Kevin Donoghue	2	/ 6
105	TuTh 1230-2pm	61 Evans	James McIvor	3	/ 6
106	TuTh 1230-2pm	55 Evans	Adam Merberg		
107	TuTh 2-330pm	61 Evans	James McIvor	4	/ 5
108	TuTh 2-330pm	55 Evans	Shamil Shakirov	5	/ 6
109	TuTh 330-5pm	39 Evans	Adam Merberg		
110	TuTh 330-5pm	47 Evans	Markus Vasquez		
111	TuTh 5-630pm	47 Evans	Markus Vasquez		
112	TuTh 5-630pm	122 Latimer	Shamil Shakirov		
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Other/none, explain: _____

Instructions:

- Closed book: No notes, no books, no calculators.
- Exam time 50 minutes, do all of the problems.
- You must justify your answers for full credit.
- Write your answers in the space below each problem.
- If you need more space, use reverse side or scratch pages. Indicate clearly where to find your answers.

1. (6 points) Determine if each of the following series converge.

a)
$$\sum_{n=1}^{\infty} \frac{(n+1) \cdot 3^n}{2^{2n+1}}$$

b)
$$\sum_{n=1}^{\infty} \frac{2^{n+1} + (-3)^n}{4^n}$$

c)
$$\sum_{n=1}^{\infty} \frac{2n^2}{n^2 + 4n + 3}$$

2. (6 points) Compute each of the following integrals.

a) $\int_1^9 \frac{3x - 2}{2\sqrt{x}} dx$

b) $\int_{-1}^0 (x + 1)e^{-x} dx$

c) $\int_0^{\infty} \frac{e^x}{(1 + e^x)^2} dx$

3. (6 points) Compute each of the following integrals.

a) $\int \frac{3x}{(x+2)^{1/4}} dx$

b) $\int_{-\pi/2}^{\pi/2} (x + \cos x) \cos(\sin x) dx$

4. (5 points) Consider the integral $\int_0^1 \frac{1}{1 + \cos^2(\pi x)} dx$.

a) Approximate the integral using the trapezoid rule with $n = 4$ subintervals (simplify to a rational number).

b) Estimate how many times smaller the error in your approximation would be if you instead used $n = 20$ subintervals. *Hint:* Consider how the error bound formula $\frac{C(b-a)^3}{12n^2}$ depends on n .

5. (6 points) Consider the function $f(x) = \begin{cases} C \cdot \sin(\pi x) & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$

a) Find all numbers C that make f the pdf of a continuous random variable X .

b) Find the cdf of X .

c) Find the probability $P(X \geq \frac{1}{3})$.