UC Berkeley Math 10A, Fall 2014: Final Exam

Prof. Persson, December 15, 2014

Name: ______________________________

SID: ______________________________

Section: Circle your discussion section below:

Sec	Time	Room	GSI
101	TuTh 8-930am	35 Evans	Noble Macfarlane
102	TuTh 8-930am	31 Evans	Kevin Donoghue
103	TuTh 11-1230pm	45 Evans	Noble Macfarlane
104	TuTh 11-1230pm	41 Evans	Kevin Donoghue
105	TuTh 1230-2pm	61 Evans	James McIvor
106	TuTh 1230-2pm	55 Evans	Adam Merberg
107	TuTh 2-330pm	61 Evans	James McIvor
108	TuTh 2-330pm	55 Evans	Shamil Shakirov
109	TuTh 330-5pm	39 Evans	Adam Merberg
110	TuTh 330-5pm	47 Evans	Markus Vasquez
111	TuTh 5-630pm	47 Evans	Markus Vasquez
112	TuTh 5-630pm	122 Latimer	Shamil Shakirov

Other/none, explain: ______________________________

Grading

1	/ 3
2	/ 3
3	/ 7
4	/ 6
5	/ 9
6	/ 9
7	/ 4
8	/ 3
9	/ 6
10	/ 8
11	/ 7
	/65

Instructions:

- Closed book: No notes, no books, no calculators.
- Exam time 3 hours, do all of the problems.
- You must justify your answers for full credit.
- Write your answers in the space below each problem.
- If you need more space, use reverse side or scratch pages. Indicate clearly where to find your answers.

1. (3 points) Find the derivative of $f(x) = x^x$. *Hint*: Use exponent rules.

2. (3 points) Find all values a, if any, where the tangent line to $f(x) = \dfrac{x+1}{x-1}$ at $x = a$ is parallel to the line $y = x - 1$.

3. Consider the function $f(x) = 2x^2 - x^4$.

a) (4 points) Find all critical points of $f(x)$, and decide which, if any, are local maxima and which are local minima.

b) (3 points) Find the absolute minimum and the absolute maximum of $f(x)$ on the interval $[0, 2]$.

4. Consider the probability density function (pdf) $f(x) = \frac{b}{2}e^{-b|x|}$, where $b > 0$.

a) (2 points) Find the expected value of a random variable with pdf $f(x)$.

b) (4 points) Suppose that $x_1, ..., x_n$ form a random sample from a distribution with pdf $f(x)$. Find the maximum likelihood estimate of the parameter b.

5. Compute each of the following integrals.

a) (3 points) $\displaystyle\int_1^8 \frac{1}{x^{2/3}}\,dx$

b) (3 points) $\displaystyle\int x^3 \sin(-x^2)\,dx$

c) (3 points) $\displaystyle\int_{-2}^{2} \sqrt{4 - x^2}\,dx$ *Hint*: Consider what the integral represents.

6. Determine if the following series converge or diverge.

a) (3 points) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{1+e^n}$

b) (3 points) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2 \cdot 2^{3n}}{3^{3n+1}}$

c) (3 points) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{1+e^{-n}}$

7. (4 points) Suppose that five fishermen are fishing together, and the number of fish each of them catches in a day is a Poisson random variable with $\lambda = 5$, which is independent across all of the fishermen. What is the probability that exactly two of the five fishermen will catch no fish on a given day?

8. (3 points) Let a be any positive real number and let $f(x) = \dfrac{1}{x} - a$. Apply Newton's method to the function $f(x)$ to derive an iterative formula of the form $x_{n+1} = g(x_n)$ for estimating $1/a$. Simplify your expression for $g(x_n)$ so it does not involve any divisions.

9. Consider the function $f(x) = \dfrac{Ce^x}{1+e^{2x}}$.

a) (4 points) Find all numbers C that make f the pdf of a random variable.

b) (2 points) Find the cdf of f for the value of C that you found in **(a)**.

10. Solve the following problems using the table below, which shows values for the integral of the standard normal distribution $f(x)$:

a	0.0	0.5	1.0	1.5	2.0	2.5	3.0
$\int_{-\infty}^{a} f(x)\, dx$	50%	69.1%	84.1%	93.3%	97.7%	99.4%	99.9%

a) (2 points) For X normal distributed with mean $\mu = 1$ and standard deviation $\sigma = 1$, calculate $P(X \geq 0)$.

b) (2 points) For X normal distributed with mean $\mu = -2$ and standard deviation $\sigma = 2$, calculate $P(-3 \leq X \leq -1)$.

c) (4 points) To determine the effectiveness of a certain diet in reducing the amount of cholesterol in the bloodstream, 100 people are put on the diet. After they have been on the diet for a sufficient length of time, their cholesterol count will be taken. The nutritionist running this experiment has decided to endorse the diet if at least 65 percent of the people have a lower cholesterol count after going on the diet. Use the central limit theorem to estimate the probability that the nutritionist endorses the diet if, in fact, it has no effect on the cholesterol level.

11. The Taylor series for $f(x) = \arctan x$ centered about $x = 0$ is $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$.

a) (4 points) Find the radius of convergence of this Taylor series.

b) (3 points) Find the Taylor series for $g(x) = x \cdot \arctan x^2$.