MSRI–Evans Talk

Monday, 4:10–5:00pm, 60 Evans

Feb. 6 **Jean-Louis Colliot-Thélène**, CNRS and Clay Mathematical Institute, MSRI

Integral solutions of polynomial equations

Necessary conditions for the existence of integral solutions of a polynomial equation $f(x_1, \ldots, x_n) = 0$ are provided by congruence conditions together with the condition that a solution should exist over the reals. When f is a quadratic form, it is a classical result that these conditions, are sufficient. For f an arbitrary polynomial, I will give several examples to show that these conditions are not sufficient. Appeal here is made to the law of quadratic reciprocity. I will then explain how the examples with homogeneous forms (or more generally projective varieties) can be interpreted by means of the Brauer–Manin obstruction. I will show how the examples with nonhomogeneous forms (over the integers) can be interpreted by means on an integral variant of the Brauer-Manin obstruction.

I will then go on to describe cases where the Brauer-Manin obstruction is the only obstruction to the existence of integral solutions.

I shall first describe an algorithm to solve the classical question whether an integer is represented by an integral, indefinite, ternary quadratic form.

I will then discuss rational points of algebraic varieties : review of the situation for curves; rational surfaces; surfaces with a pencil of curves of genus one; intersections of two quadrics in arbitrary dimension. Some of the results here are conditional on the finiteness of Tate-Shafarevich groups and on the Schinzel hypothesis.