Math 115 | Professor K. A. Ribet
First Midterm Exam | September 23, 1999

This is a closed-book exam: no notes, books or calculators are allowed. Explain your answers in complete English sentences. No credit will be given for a "correct answer" that is not explained fully.

1 *(4 points)*. Find the remainder when 2^{33} is divided by 31.

2 *(4 points)*. Use the identity $27^2 - 8 \cdot 91 = 1$ to find an integer x such that $27x = 14 \bmod 91$.

3 *(4 points)*. Find all prime numbers p such that $p^2 + 2$ is prime.

4 *(5 points)*. Suppose that $ax + by = 17$, where a, b, x and y are integers. Show that the numbers $\gcd(a, b)$ and $\gcd(x, y)$ are divisors of 17. Decide which, if any, of the following four possibilities can occur:

(i) $\gcd(a, b) = \gcd(x, y) = 1$;
(ii) $\gcd(a, b) = 17$ and $\gcd(x, y) = 1$;
(iii) $\gcd(a, b) = 1$ and $\gcd(x, y) = 17$;
(iv) $\gcd(a, b) = \gcd(x, y) = 17$.

5 *(6 points)*. Suppose that n is composite: an integer greater than 1 that is not prime. Show that $(n - 1)!$ and n are not relatively prime. Prove that the congruence $(n - 1)! \equiv -1 \bmod n$ is false.

6 *(6 points)*. Prove that -1 is not a square modulo the prime p if $p \equiv 3 \bmod 4$.

7 *(6 points)*. Show that $x^8 \equiv 1 \bmod 20$ if x is an integer that is prime to 20. Find the integer t such that $t^9 = 760231058654565217 \approx 7.60231 \times 10^{17}$.