George M. Bergman
10 Evans Hall

Fall 1999, Math 1B

First Midterm

21 September, 1999
8:10-9:30 AM

1. (30 points, 6 points apiece) Find the following.

(a) $\int (x+1)\, e^x \, dx$

(b) $\int \sin^3 x \, \cos^3 x \, dx$

(c) $\int_{-1}^{1} \frac{(x+2)^2}{x^2+1} \, dx$

(d) An integral expressing the length L of the curve $y = \sin x$ from $x = a$ to $x = b$. Do not attempt to carry out the integration.

(e) $\lim_{n\to\infty} ((n + n^{-1})^2 - n^2)$

2. (40 points, 10 points apiece) Compute the following integrals.

(a) $\int \sin \sqrt{x} \, dx$

(b) $\int \frac{dx}{4\,x^{2/3} - 4\,x^{1/3} - 3}$

(c) $\int (6\,x - x^2)^{-1/2} \, dx$

(d) $\int_0^{e^{-2}} t^{-1}(\ln t)^{-2} dt$

3. (12 points) (a) (6 points) If f is a continuous function on the real line, what is meant by $\int_0^\infty f(x)\, dx$ (assuming this exists)?

(b) (6 points) Let f be a function such that $\int_0^\infty f(x)\, dx$ exists. Let us call its value L, and let c be any positive real number. Derive from the definition a formula expressing $\int_0^\infty f(cx)\, dx$ in terms of L and c. (Correct reasoning: 3 points; correct formula: 3 points.)

4. (18 points) (a) (9 points) State the Principle of Mathematical Induction.

(b) (9 points) Recall that the *Fibonacci numbers* are the sequence of numbers $f_1, f_2, f_3, \ldots$ defined by $f_1 = f_2 = 1$, and $f_{n+1} = f_n + f_{n-1}$ for $n \geq 2$. Prove that for all $n \geq 1$, $f_{n+1}^2 - f_{n+1} f_n - f_n^2 = (-1)^n$. (Suggestion: use Mathematical Induction. In proving S_{k+1} from S_k, apply the formula saying each Fibonacci number is the sum of the two preceding to the highest Fibonacci number occurring.)