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2050 VLSB

Fall 1998, Math 1AW
Second Midterm

6 November, 1998
3:10-4:00 PM

1. (50 points, 10 points apiece) Find the following.

(a) $\lim_{x \rightarrow 1} \frac{e^{2x} - e^{x+1}}{x-1}$.

(b) $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{1 - \cos 3x}$.

(c) An antiderivative of the function $x + x^{-1}$

(d) A function satisfying the differential equation $f'(x) = -9f(x)$, and such that $f(1) - f(0) = 1$

(e) $\frac{d}{dx} 1 + x^{\cosh x}$

2. (25 points) (a) (10 points) State the principle of mathematical induction.

(b) (15 points) Suppose f is an infinitely differentiable function (i.e., a function such that $f', f'', \dots, f^{(n)}, \dots$ all exist). Prove that for all positive integers n , one has $D^n(x f(x)) = x f^{(n)}(x) + n f^{(n-1)}(x)$. Here $f^{(0)}$ means f . Suggestion: Use mathematical induction.

3. (25 points) (a) (15 points) Give the information asked for below about the curve $y = \ln \frac{x^2 + 1}{5}$.

If any of the items asked for does not exist, write "None". (For limits, write "None" only if the function does not approach either a real number or $\pm \infty$.)

(b) (10 points) Sketch the curve. Your sketch does not need to reflect accurate numerical values of the coordinates of the various transition points, as long as it correctly shows the order in which these occur.

x -intercept(s): _____ y -intercept: _____ increasing on interval(s): _____ decreasing on interval(s): _____ concave up on interval(s): _____ concave down on interval(s): _____ vertical asymptote(s): _____ $\lim_{x \rightarrow \infty} y =$ _____ $\lim_{x \rightarrow -\infty} y =$ _____ extrema: _____