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Fall 1996, Math 53

First Midterm – makeup version

2 Oct., 1996
10:10-11:00 AM

1. (25 points) Let C be the curve given by the parametric equations $x = \sin\theta$, $y = 2\cos\theta$, $0 \leq \theta < \pi/2$.

(a) (13 points) Describe C by an equation expressing y as a function of x, with restrictions on the values of x if necessary, and sketch the curve.

(b) (12 points) Find an equation for the line that is tangent to C at the point having parameter θ.

2. (25 points) Let f be a continuously differentiable real-valued function on the interval $[0, 2\pi]$. Show that the space curve given by the parametric equations $x = f(t)$, $y = \sin t$, $z = \cos t$, $0 \leq t \leq 2\pi$ has the same arc-length as the plane curve $y = f(x)$, $0 \leq x \leq 2\pi$. You may assume formulas for arc length given in Stewart.

3. (25 points) Calculate the area of the surface gotten by rotating about the x-axis the plane curve

$$x = t - (t^3/3), \qquad y = t^2, \qquad 0 \leq t \leq 1.$$

4. (25 points) Suppose $f(x, y)$ is a real-valued function of two variables, defined for all real numbers x and y.

(a) (7 points) Given a point (x_0, y_0), and a real number L, define what it means for $\lim_{(x,y)\to(x_0,y_0)} f(x, y) = L$ to hold.

(b) (7 points) Define what it means for f to be continuous at (x_0, y_0).

(c) (11 points) Let f be the function defined by the formulas $f(x, y) = x$ if $x^2 + y^2 \leq 1$, $f(x, y) = y$ if $x^2 + y^2 > 1$. At what points is f continuous, and at what points is it discontinuous? The answer will involve more than one case; explain the reason for at least *one* case of your answer.