Math 55: First Midterm, 26 September 1996

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Problem 0: Write your name, your section time and number, and your TA's name on the cover of your blue book. Books, notes, calculators, scratch paper and/or collaboration are not allowed.

Problem 1: Prove that $(p \wedge q) \rightarrow p$ is a tautology.

Problem 2: Let f, g and h be defined by:

$$f : \mathbf{R} \rightarrow \mathbf{R}, \qquad f(x) = x^3$$

$$g : \mathbf{Z} \rightarrow \mathbf{Z}, \qquad g(n) = n^3$$

$$h : \mathbf{R} \rightarrow \mathbf{Z} \times \mathbf{Z}, \qquad h(x) = (\lfloor x \rfloor, \lceil x \rceil)$$

For each of the functions f, g and h, state whether the function is 1-1, whether it is onto, and whether it is invertible.

Problem 3: Show that

$$\sum_{k=1}^{n} k^2 = O(n^3).$$

Problem 4: Construct pseudocode for an algorithm which accepts input consisting of two finite sets $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_m\}$ and a function $f : A \rightarrow B$, and returns output T if f is onto and F if f is not onto.

Problem 5: Suppose $a \equiv b$ and $c \equiv d$ mod 17. Show that $ac \equiv bd$ mod 17.

Problem 6: Use the Euclidean algorithm to compute gcd(277, 123).

Problem 7: Suppose x is an integer with $0 \leq x \leq 1000$ and

$$x \bmod 7 = 3, \qquad x \bmod 11 = 5, \qquad x \bmod 13 = 7.$$

(a) Is x uniquely determined by this information? Why or why not?
(b) Calculate x^2 mod 7 and x^3 mod 11.