Math 113: Introduction to abstract algebra.

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Fall 1995, MWF 11–12 a.m., 2 Evans.

Required text:

John B. Fraleigh, *A first course in abstract algebra*, fifth edition, Addison-Wesley Publishing Company, Reading, Mass., 1994.

Syllabus for the midterm, Wednesday, October 18, 11-12 a.m., 2 Evans:

Chapter 0; Chapter 1 (without the subsections *Finite-state machines* and *Cayley digraphs*); Chapter 2, pp. 87–136 (that is, sections 1–3 and section 4 until the subsection *Periodic functions*); Chapter 3, sections 1–4; and, as treated in class, the quaternions of Hamilton (bottom p. 336–top p. 338), as well as the quaternion group $Q = \{1, i, j, k, -1, -i, -j, -k\}$ of order 8 (called G on the middle of p. 338).

The exam is open book, open notes, no calculators. No blue books are required. A sample midterm follows.

Name:

Note. You have to do **three** out of the four problems. Cross out the problem that you don't want to be graded. Theorems proved in the book or in class may be used without proof (but do give the formulation).

1	
2	
3	
4	
Total	

Problem 1.

Find an element of order 30 in the symmetric group S_{10}. Is the element that you found even or odd? Explain your answer.

Solution:

Problem 2.
Let $G = \mathbb{Z}_3 \times \mathbb{Z}_{10}$, and let a be the element $(2,5)$ of G. Compute the order of a as well as the index $(G : \langle a \rangle)$. Is $G/\langle a \rangle$ cyclic? Explain your answer.

Solution:

Problem 3.
Denote by $\mathrm{GL}(2,\mathbb{C})$ the group of invertible 2×2 matrices with complex entries, and write $N = \{A \in \mathrm{GL}(2,\mathbb{C}) \mid |\det A| = 1\}$. Prove that N is a normal subgroup of $\mathrm{GL}(2,\mathbb{C})$, and that the factor group $\mathrm{GL}(2,\mathbb{C})/N$ is isomorphic to the multiplicative group $\mathbb{R}^*_{>0}$ of positive real numbers.

Solution:

Problem 4.
Let G be a *finite* group, and let $f: G \to \mathbb{Z}$ be a group homomorphism. Prove that $f(g) = 0$ for all $g \in G$.

Solution: