Department of Mathematics, University of California, Berkeley

Math 53

Alan Weinstein, Fall 2005

Second Midterm Exam, Tuesday, November 8th 2005

Instructions. BE SURE TO WRITE YOUR NAME AND YOUR GSI'S NAME ON YOUR BLUE BOOK. Read the problems very carefully to be sure that you understand the statements. All work should be shown in the blue book; writing should be legible and clear, and there should be enough work shown to justify your answers. **Indicate the final answers to problems by circling them.** [Point values of problems are in square brackets. The total point value is 45, for 15% of your course grade.]

PLEASE HAND IN YOUR SHEET OF NOTES ALONG WITH YOUR BLUE BOOK. YOU SHOULD ***NOT*** HAND IN THIS EXAM SHEET.

1. [12 points] All that I'm going to tell you about the differentiable function f on $\mathbb{R}^2$ is that its gradient vector at $(2,3)$ is $4\mathbf{i}-\mathbf{j}$, and that $f(2,3)=7$. Answer those of the following questions for which enough information is given, and explain why not enough information is given for the others.
(a) Is $(2,3)$ a local maximum or minimum point (or neither) for f?
(b) Find the best approximation you can for $f(2.04, 2.99)$.
(c) There is one point on the graph of f where you have enough information to find the equation of the tangent plane. Write down the (three) coordinates of that point, and the equation of that tangent plane.
(d) Is $f(2,4)$ greater or less than 7?
(e) Find $g'(1)$, where $g(t)=f(2t^2,t^3)$.

2. [10 points]
(a) Sketch the region of integration D for the following iterated integral.

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dx\,dy$$

(b) Evaluate the integral by reversing the order of integration.
(c) Find an integer n such that the average value of the function $\sqrt{x^3+1}$ over the region D lies between n and $n+1$.

3. [13 points] Let $A(q)$ be the area of the region R_q defined by $x\geq 0$, $y\geq 0$, and $\sqrt{x}+\sqrt{y}\leq q$.
(a) Find a number p, depending on q, so that the transformation $x=pu$, $y=pv$ takes the region S in the (u,v)-plane defined by $u\geq 0$, $v\geq 0$, and $\sqrt{u}+\sqrt{v}\leq 1$ to the region R_q in the (x,y)-plane.
(b) Use the transformation in (a) to write $A(q)$ as a double integral over the region S.
(c) Using your result in (b), tell what happens to $A(q)$ when q is doubled.
(d) Use the change of variables $x=r^2$, $y=s^2$ to find $A(q)$.

4. [10 points]
(a) A rectangular box *with no top* is to have a surface area of 48 square meters. Use the method of Lagrange multipliers to find the dimensions which maximize the volume.
(b) **Extra credit.** Using the fact that a cube maximizes volume among all rectangular parallelepipeds with a given surface area, solve part (a) without using any calculus.