Math 1B, Final Examination
N.Reshetikhin, December 13, 2005

Problem	1	2	3	4	5	6	7	8	9	10	11	12	*Total*
Points	10	15	15	15	15	15	15	15	15	10	15	20	175
Grade													

Student's Name:

GSI's name:

Student's i.d. number:

1.(*10 pnts*) Evaluate the integral

$$\int \frac{\sin(x^{\frac{1}{3}})}{x^{\frac{1}{3}}} dx$$

2.(*15 pnts*) Evaluate the integral

$$\int \frac{1}{(t^2-4)(t-1)}dt$$

3.(*15 pnts*) Indicate which of the following statements are true and which are false. Do not show your work.

1. $\int_1^\infty \frac{\cos^2 x}{x^{\frac{1}{3}}} dx$ converges by comparison test with $\int_1^\infty \frac{1}{x^{\frac{1}{3}}} dx$.

2. $\int_1^\infty \frac{\ln x}{x} dx$ diverges by comparison test with $\int_1^\infty \frac{1}{x} dx$.

3. $\int_1^2 \frac{dx}{\sqrt{x-1}} dx$ is a divergent improper integral.

4. $\int_{-\infty}^\infty \frac{1}{(x-1)^{\frac{3}{2}}} dx$ is a divergent improper integral.

5. $\int_0^\infty \frac{\ln(x+1)}{x^{3/2}} dx$ is a convergent improper integral.

4.(*15 pnts*) Find the radius and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} n^2 \left(\frac{x-2}{3}\right)^n$$

5.(*15 pnts*) State whether the following series is absolutely convergent, conditionally convergent, or divergent. Do not show your work.

1. $\sum_{n=1}^{\infty} \frac{2+\sin(n^2)}{n}$.

2. $\sum_{n=1}^{\infty} (-1)^n \frac{\sin^2 n + n}{13+4n^2}$.

3. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n}} \right)$.

4. $\sum_{n=1}^{\infty} \frac{10^n}{n!} (-1)^n$.

5. $\sum_{n=2}^{\infty} \cos(\pi n(n+1)) \frac{1}{n \ln(n)}$

6.(*15 pnts*) For each statement indicate whether it is true or false. Do not show your work.

1. If $\sum_{n=1}^{\infty} c_n$ converges, then $\sum_{n=1}^{\infty}(-1)^n c_n$ also converges.

2. If $f(x)$ is twice differentiable, $f'(x) > 0$, $f''(x) < 0$ and $\lim_{x\to\infty} f(x) = 2$, then the series $\sum_{n=1}^{\infty} f'(n)$ converges.

3. If the sequence $\{a_n\}$ converges and the sequence $\{b_n\}$ converges then $\{\frac{a_n}{b_n}\}$ converges.

4. If the sequence $\{a_n\}$ converges to zero, the sequence $\{b_n\}$ diverges then $\{a_n b_n\}$ diverges.

5. If $\sum_{n\geq 0} a_n 2^n$ converges and $\sum_{n\geq 0} a_n(-6)^n$ diverges, then $\sum_{n\geq 0} a_n$ diverges.

7.(*15 pnts*) Find the Taylor series of the function

$$f(x) = \sqrt{x-1}$$

about the point $x = 2$.

8.(*15 pnts*) Solve the initial-value problem.

$$(x+1)^2y' = (1+y)^2 \ , y(0) = 1$$

9.(*15 pnts*) **Find the general solution to the differential equation**

$$\frac{dy}{dx} = -(\sin x)y + \frac{1}{2}\sin 2x, \ .$$

10.(*10 pnts*) Find the general solution to the differential equation

$$y'' - y = x \sin x.$$

11.(*15 pnts*) Match pictures to differential equations.

1. $\frac{dy}{dx} = y^2 + x^2$ 2. $\frac{dy}{dx} = y + x$ 3. $\frac{dy}{dx} = y^{-1}$ 4. $\frac{dy}{dx} = y^{-1} - x$ 5. $\frac{dy}{dx} = xy$

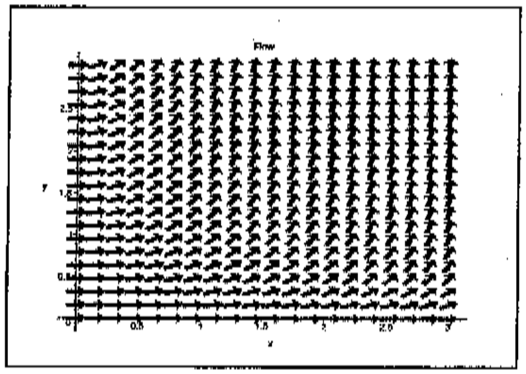

Figure 1: Equation number ....

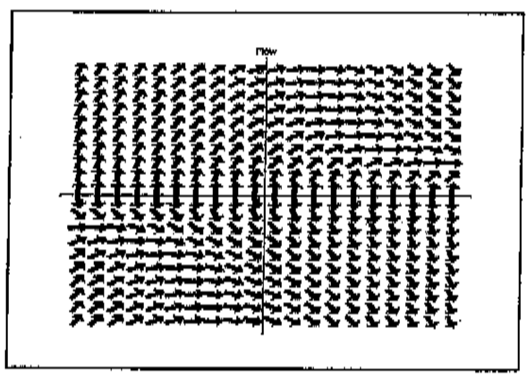

Figure 2: Equation number ....

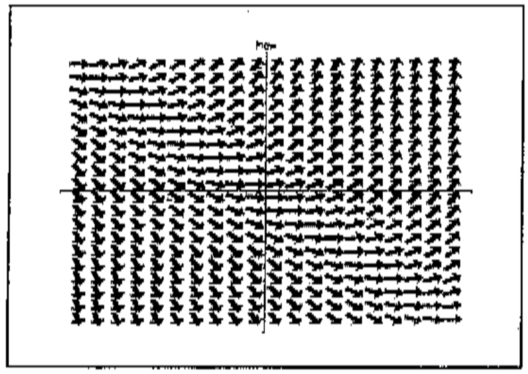

Figure 3: Equation number ....

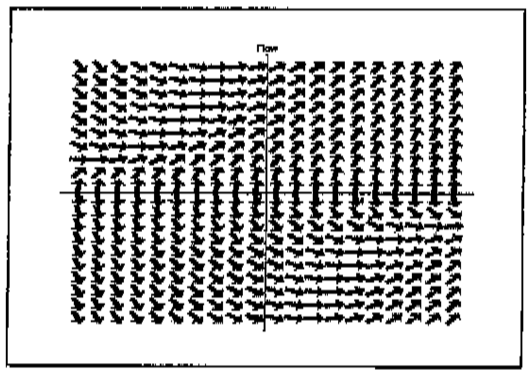

Figure 4: Equation number ....

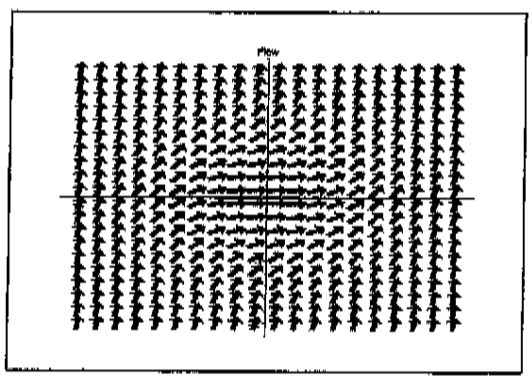

Figure 5: Equation number ....

12.(*20 pnts*) Find the power series solution to the differential equation:

$$y'' - 4xy' - 4y = 0, \quad y(0) = 1, \quad y'(0) = 0 \ .$$