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Math 113 Second Midterm

50 minutes

1. (20 points) Carefully define the following. (In each definition you may use without defining them any terms or symbols that were used in the text prior to that definition.)

   (a). Alternating group on n letters

   (b). Cycle

   (c). Factor group

   (d). Center of a group

   (e). Division ring

2. (30 points) (a). Write the following permutation as a product of disjoint cycles:

$$(1,2,4)(3,4,5)(1,5)$$

   (b). Classify the following group according to the fundamental theorem of finitely generated abelian groups.

$$(\mathbb{Z}_{100} \times \mathbb{Z}_{100})/\langle(6,6)\rangle$$

   (c). Find all solutions in $\mathbb{Z}$ of the congruence

$$6x \equiv 33 \pmod{15}$$

3. (15 points) Let S be a subset of a group G. Prove that there is a smallest normal subgroup of G containing S.

4. (15 points) Let H be a subgroup of a group G. We can think of G as an H-set via the action of left translation.

   (a). Determine the orbits of this group action.

   (b). For each $x \in G$, determine the isotropy subgroup of x.

5. (20 points) Let R be a finite ring with $1 \neq 0$. Prove that every nonzero element of R is a zero divisor or a unit.