Math 128a Midterm Exam 2

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October 24, 2003

NAME (printed) : _____
(Last Name) (First Name)

Signature : _____

Student Number : _____

(1) Do NOT open this test booklet until told to do so

(2) Do ALL your work in this test booklet

(3) Show ALL your work

(4) Check that there are 6 problems and 7 pages (including this one)

(5) NO CALCULATORS

(6) Please keep your arms and legs inside the ride at all times.

1	2	3	4	5	6	TOTAL

1 a: (4 pts) Consider

$$A = \begin{bmatrix} 1 & 4.25 & 1.25 \\ 4 & 1 & 1 \\ 1 & 1.25 & 4.5 \end{bmatrix}$$

Use Gaussian elimination, with partial pivoting to compute the determinate of A.

b: (3 pts) If it takes 10 seconds to compute the determinate of a random 1000×1000 matrix, how long would it take to compute the determinate of a random 5000×5000 matrix?

2 a: (4 pts) A matrix A is positive definite if $x^t Ax > 0$ for all $x \neq 0$. Prove that the diagonal entries $a_{i,i} > 0$

b: (3 pts) Find $\alpha > 0$ such that the following system is strictly diagonally dominate.

$$\begin{bmatrix} \alpha & 2 & 3 \\ 10 & 20 & 7 \\ \alpha & 3 & 10 \end{bmatrix}$$

3 a: (4 pts) Let A be a $n \times n$, non-singular lower triangular matrix. How many step of the Jacobi Iterative method are needed to solve $Ax = b$? (Justify your answer.)

b: (4 pts) Compute the first two steps of the Jacobi Iterative method, with starting point (0,0), to the system

$$\begin{bmatrix} 10 & 3 \\ 2 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

4 a: (4 pts) Given $f(-1) = -4$, $f(0) = -3$ and $f(1) = 0$, use Neville's method to approximate $f(2)$.

b: (5 pts) Use a variation of the Newton Divide Difference method for Hermite polynomials to find the unique polynomial, of degree at most three, such that

$$P(-1) = -4, P(0) = -1, P'(0) = 2, P(1) = 2$$

5: (5 pts) A natural cubic spline S on $[0, 2]$ is defined by

$$S(x) = \begin{cases} x^3 & \text{if } 0 \leq x \leq 1 \\ a + b(x-1) + c(x-1)^2 + d(x-1)^3 & \text{if } 1 \leq x \leq 2 \end{cases}$$

Find a, b, c and d.

6 a: (4 pts) Complete the factorization below

$$\begin{bmatrix} 2 & 0 & -1 \\ 4 & -3 & -5 \\ -2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \_ & \_ & 0 \\ \_ & \_ & 1 \end{bmatrix} \begin{bmatrix} \_ & \_ & \_ \\ 0 & 1 & \_ \\ 0 & 0 & \_ \end{bmatrix}$$

b: (4 pts) Prove that there do not exist lower and upper triangular matrices L and U satisfying

$$\begin{bmatrix} 0 & -2 & 0 \\ 2 & 1 & 0 \\ 6 & 2 & -1 \end{bmatrix}$$