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Math 113 First Midterm

50 minutes

1. (20 points) Carefully define the following. (In each definition you may use without defining them any terms or symbols that were used in the text prior to that definition.)

   (a). Function $f\colon X \to Y$.

   (b). The inverse of a function $f\colon X \to Y$.

   (c). Group.

   (d). Cyclic subgroup.

   (e). The intersection of a collection of sets.

2. (14 points)

   (a). Solve for x, in an arbitrary group G:

   $$ab^2xc^{-2} = a^2da\ .$$

   (b). Determine (in a more concise form) the subgroup H of $\mathbb{Z}$ given by $H = \langle 30, 50, 75\rangle$.

3. (14 points)

   (a). Let S be an associative binary algebraic structure with two-sided identity e. Show that if an element $x \in S$ has a left inverse x' and a right inverse x'', then $x' = x''$.

   (b). Let S and S' be binary algebraic structures.
   Let $x \in S$ be an element with the property that $xy = yx$ for all $y \in S$.
   Show that if $\phi\colon S \to S'$ is an isomorphism of binary algebraic structures, then $x' = \phi(x) \in S'$ must have the property that $x'y' = y'x'$ for all $y' \in S'$.

4. (7 points) Draw a Cayley digraph of the group $\mathbb{Z}_5$ with generating set $\{1, 3\}$.

5. (20 points) Let G be a cyclic group with generator a.
   Show that if G has finite order n, then G is isomorphic to $\langle \mathbb{Z}_n, +_n\rangle$.