Math H53 Midterm Exam 1

October 3rd, 2003

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1 a: (5 pts) Let $f(t) = \langle 3t^2, 2t^3 \rangle$. Find the length of $f(t)$ between $0 \leq t \leq 2$.

b: (5 pts) Find all solutions to $z^4 = -4$.

2: (5 pts) Let f be a harmonic function with continuous partial derivatives of any order. Further let $f_x(x, y) = 2x + y$. Find $f_y(x, y)$.

3: (5 pts) Let $r(\theta) = 4\sin(3\theta)$. Find the area of the curve in one loop.

4 a: (5 pts) Consider the function $f(x, y) = x^4 + y^4 + x^2y^2 - xy + 3$. Find the direction of steepest descent from the point $(1, 2)$

b: (5 pts) Given

$$z^2 + 4x + 4y^2 - 24y = x^2 + 2z$$

Convert this to standard form. What sort of quadratic surface is this. (If you can't remember the name, just draw a picture)

5: (5 pts) Let V be the vector space of polynomials. Define the dot product (inner product) between two vectors f and g as

$$\int_0^1 f(x)g(x)dx$$

Find $g(x)$ orthogonal to $f(x) = x$.

6: (5 pts) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a continuous functions, with continuous partial derivatives. Let $\{(x_i, y_i)\}_{i=0}^{\infty}$ be a sequence of local maximums. Further let $\lim_{i\to\infty} x_i = c$ and $\lim_{i\to\infty} y_i = d$. Show that f has a critical point at (c, d).
Bonus: (2 pts) Give an example to show that (c, d) can be a local minimum.