Math 104, Section 6
Sarason

1ST MIDTERM EXAMINATION

Open book, open notes. In your proofs, you may use any results from the lectures, from Sections 1–12 of the textbook, or from the homework exercises. Each problem is worth 10 points.

1. Give examples of

   (a) a countable subset S of $\mathbb{R}$ such that $\sup S$ is not in S;

   (b) a divergent sequence in $\mathbb{R}$ that has exactly one cluster point.

2. Prove that every open interval (a, b) in $\mathbb{R}$ is a countable union of open intervals with rational endpoints.

3. Prove that every uncountable subset of $\mathbb{R}$ has a limit point.

4. Let $(s_n)_1^\infty$ be a sequence in $\mathbb{R}$, let C be the set of cluster points of $(s_n)_1^\infty$, and let c be a limit point of C. Prove that c is in C.

5. Let m be a natural number, $m > 1$. Let the sequence $(r_n)_1^\infty$ be defined recursively by

$$r_1 = 1, \ r_{n+1} = 1 + \frac{m-1}{1+r_n} \ (n = 1, 2, \dots).$$

   Prove that $(r_n)_1^\infty$ converges, and find its limit.