F'03 KRAUSS

MATH 110-6 - Final Exam

(1) 1. Find the Jordan form of $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

2. Find out whether $(0,0,0)$ is a local minimum or a local maximum, or neither, for the function $f(x,y,z) = -x(x+z) - y^2 - 2z^2 + x^6 + y^6$.

(2) Let V be a finite-dimensional inner product space over $\mathbb{C}$, and T a normal operator on V

Prove: 1. $\|T^* v\| = \|Tv\| \ \forall v \in V$ where $\|\cdot\|$ is the norm induced by the inner product.

2. W is the eigenspace of T associated to λ iff W is the eigenspace of T^* associated to $\bar{\lambda}$ (explain first why T and T^* have eigenvalues).

3. Two eigenvectors of T associated to different eigenvalues are orthogonal.

(3) Let W be the subspace of $\mathbb{R}^2$ spanned by the vector $[3\ 4]^T$ and P be the orthogonal projection of $\mathbb{R}^2$ with the standard inner product onto W.

Find: 1. a formula for $P(x)$, $x \in \mathbb{R}^2$.

2. the matrix of P in the standard ordered basis of $\mathbb{R}^2$.

3. $W^\perp$

4. an orthonormal basis of $\mathbb{R}^2$ in which P is represented by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.