

MATH 185 - FINAL EXAM - FALL 2003 D. Geba

① Find the Maclaurin series for the function

$$f(z) = \frac{1}{(1+z^2)^2}$$

② Find the Laurent series for the function

$$f(z) = \frac{z^3}{(z+1)(z-2)} \text{ in the domain } 0 < |z+1| < 3.$$

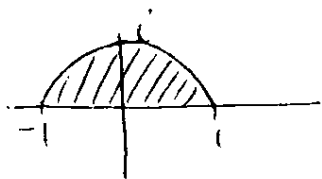
③ Find $\text{Res}_{z=0} \frac{\tan z - z}{(1 - \cos z)^2}$.

④ How many roots does the equation $z^4 - 9z + 1 = 0$ have in the annulus $1 < |z| < 2$?

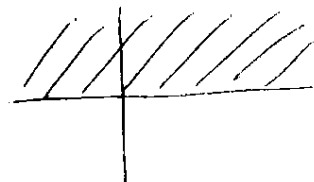
⑤ Compute the integral $\int_0^{2\pi} \frac{\cos u \varphi}{1 - 2a \cos \varphi + a^2} d\varphi$ where $u \geq 0$ is an integer and $a \in \mathbb{R}$, $-1 < a < 1$.

⑥ Find a conformal mapping $w: \Delta \rightarrow \Delta$, where $\Delta = \{ |z| \leq 1 \}$, such that $w(i) = i$ and $w\left(\frac{i}{2}\right) = \frac{4i}{5}$.

⑦ Find a conformal mapping from



onto the half-plane



⑧ Let $f: \mathbb{C} \rightarrow \mathbb{C}$ an entire function such that $\forall z \in \mathbb{C} \setminus \mathbb{R}$, $|f(z)| \leq \frac{1}{\| \text{Im } z \|}$. Prove that $f \equiv 0$.