

MAT 110, SECTION 6, MIDTERM

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① Solve the following system using Gauss elimination:

$$\begin{cases} 3x_1 - x_2 + 2x_3 + 4x_4 + x_5 = 2 \\ x_1 - x_2 + 2x_3 + 3x_4 + x_5 = -1 \\ 2x_1 - 3x_2 + 6x_3 + 9x_4 + 4x_5 = -5 \\ 7x_1 - 2x_2 + 4x_3 + 8x_4 + x_5 = 6. \end{cases}$$

② Let $A \in M_{3 \times 3}(\mathbb{R})$, $A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix}$.

Test A for diagonalizability, and if A is diagonalizable, find Q such that $Q^{-1}AQ$ is a diagonal matrix.

③ Let $A, B \in M_{2001 \times 2001}(\mathbb{R})$ such that $AB = -BA$. Prove that either A or B is not invertible.

④ Let $D = \begin{vmatrix} x & a & \dots & a \\ a & x & a & \dots & a \\ \vdots & & & & \\ \vdots & & & & \\ a & \dots & \dots & a & x \end{vmatrix}$ a $n \times n$ determinant

where $x, a \in \mathbb{R}$.

a) Prove that $D = (x + (n-1)a)(x-a)^{n-1}$.

b) What is the rank of matrix A , for which $D = \det A$? Discussion in terms of x and a .