Name:

Final Exam
Math 1B, Summer 2009
August 14, 2009
Hartglass

Unless otherwise stated, $y = y(x)$ is a function of the independent variable x.

Problem	Score
1	/11
2	/15
3	/15
4	/12
5	/15
6	/10
7	/12
8	/10
Total	/100

1.) (11 points) Find the general solution to the following differential equation:
$$\frac{dy}{dx} = \frac{y^2 + 1}{y} \cdot \frac{x}{x^2 + 5x + 4}$$

2.) (15 points) Find the general solution to the following differential equation.

$$y'' - 2y' + y = \frac{e^x}{x^2}$$

3.) *a*.) (10 points) Use mathematical induction to prove the following equality for n a nonnegative integer (i.e. $n = 0, 1, 2, 3, ...$):

$$\int_0^{\pi/2} \cos^{2n}(x)dx = \frac{\pi}{2} \cdot \frac{1 \cdot 3 \cdot 5 \cdot ... \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdot ... \cdot 2n}.$$

Note that if $n = 0$, the expression $\dfrac{1 \cdot 3 \cdot 5 \cdot ... \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdot ... \cdot 2n}$ is understood to be 1 (the empty product).

b.) (5 points) Use the result of part a.) to find a formula for $\int_{0}^{\infty} \dfrac{dx}{(x^2 + 4)^n}$ for n a positive integer (i.e. $n = 1, 2, 3, ...$). (You do not need to use induction here).

4.) (12 points, 4 points each) Determine if the following series are absolutely convergent, conditionally convergent, or divergent. Please clearly state what tests you are using. Note that this problem continues onto the next page.

a.) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n^2}$

b.) $\displaystyle\sum_{n=1}^{\infty} \frac{2 + \sin(n)}{\sqrt{n^2 + 3n + 4}}$

c.) $\displaystyle\sum_{n=1}^{\infty} (-1)^n n \arctan(1/n^{5/2})$

5.) (15 points, 3 points each) Determine if the following statements about sequences and series are sometimes true, always true, or never true. If your answer is *always* or *never* you do not need to justify your statement but if your answer is *sometimes*, you must provide one example where the statement holds and one example where the statement does not hold. You do not have to justify that your examples do/do not work.

a.) If $\sum_{n=1}^{N} a_n \geq -80$ for all N and $a_n \leq 0$ for all n then $\sum_{n=1}^{\infty} a_n$ is convergent.

b.) If $\sum_{n=1}^{\infty} a_n$ convergent with $a_n > 0$ for all n, then $\sum_{n=1}^{\infty} \frac{\sin(a_n^{3/2})}{a_n}$ is convergent.

c.) If $\displaystyle\sum_{n=1}^{\infty} c_n 3^n$ is conditionally convergent then $\displaystyle\sum_{n=1}^{\infty} c_n(-3)^n$ is absolutely convergent.

d.) If $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent then so is $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n a_n}{\sqrt{n}}$.

e.) If the sequence $\{a_n^2\}$ is convergent then the sequence $\{a_n\}$ is convergent

6.) (10 points) Find the interval of convergence and the radius of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{(3x-9)^{2n}}{12^n \sqrt{n}}$$

7.) (12 points, 6 points each) Find the Maclaurin series for the following functions. *Please put your answer in $\sum$ notation.*

a.) $f(x) = \int_0^x t \sin(t^4) dt$

b.) $g(x) = e^x \cos(x)$

(*Hint*: Use the exponential form of $\cos(x)$ and multiply the two functions. This will give you two power series which are not hard to find. Your answer is allowed to include complex numbers.)

Please answer **ONE** of the following two questions. If you answer both questions, please clearly indicate which question you want graded.

8A.) (10 points) a.) Show that $\left| \sqrt{e} - \left(1 + \dfrac{1}{2} + \dfrac{1}{2^2 \cdot 2!} + \dfrac{1}{2^3 \cdot 3!} \right) \right| \leq \dfrac{1}{192}$. You may use the fact that $\sqrt{e} < 2$.

b.) Find the values of p where the following series is convergent:
$$\sum_{n=1}^{\infty} \frac{1}{n^p(\sqrt{n+1} - \sqrt{n})^{3/2}}$$

8B.) (10 points) Solve the differential equation $y'' + xy' + y = 0$ subject to the initial conditions $y(0) = 3$ and $y'(0) = 0$. Your answer may include power series if you wish.