1. (20 points) Sketch the graph of $f(x) = xe^x$, labeling all relevant details.

2. (20 points) Consider the following integrals:

$$A = \int_1^4 2x \ln x \, dx \qquad B = \int_0^3 2x \ln(x+1) \, dx$$
$$C = \int_1^4 2(x-1) \ln x \, dx \qquad D = \int_0^9 \ln(\sqrt{x}+1) \, dx$$

Which of these integrals are equal to each other? Explain.

3. (10 points) Suppose the coefficients of the cubic polynomial $P(x) = a + bx + cx^2 + dx^3$ satisfy $a + \frac{b}{2} + \frac{c}{3} + \frac{d}{4} = 0$. Show that $P(x) = 0$ has a root between 0 and 1.
Hint: What is the average value of P on $[0,1]$?

4. (20 points) Find the volume of the solid generated by rotating the region bounded by the given curves around the specified axis.
   (a) $y = x^3$, $y = 0$, $x = 1$; about $x = 2$
   (b) $y = 1/x$, $x = 1$, $x = 2$, $y = 0$; about the x-axis

5. (15 points) Evaluate the indefinite integral.

(a) $\displaystyle\int \frac{x^3}{1+x^4} \, dx$ (b) $\displaystyle\int \tan x \ln\big(\cos(x)\big) \, dx$ (c) $\displaystyle\int \left(\frac{1-x}{x}\right)^2 dx$

6. (10 points) Find $f'(x)$ if $f(x) = \displaystyle\int_{\sqrt{x}}^{x} \frac{e^t}{t} \, dt$.

7. (10 points) Prove $\displaystyle\frac{1}{e} \le \int_0^1 e^{-x^2} \, dx \le 1$.

8. (15 points) Use a Riemann sum to compute $\displaystyle\int_0^1 x^2 \, dx$.

Hint: You may find it useful to know $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

9. (10 points) We say that two curves are orthogonal if their tangent lines are perpendicular at each point where the curves intersect. Show that $y = cx^2$ and $x^2 + 2y^2 = k$ are orthogonal for any c and any $k > 0$.
Hint: Two lines are perpendicular if the product of their slopes is -1.

10. (10 points) Use one iteration of Newton's method to approximate $\sqrt{8}$ using the starting approximation $x_1 = 3$.

11. (10 points) Find f given that $f''(x) = \sin x$, $f(0) = 1$, and $f'(0) = 0$.