Mathematics 1BM
First Midterm Exam

Professor K. A. Ribet
February 26, 1997

60 Evans and 2060 VLSB
9:10–10 AM

Your Name: ______________________________ TA: ________________________

This booklet should consist of a cover sheet, four pages of questions, and a final reference page of formulas. Please check that your booklet is complete, and write your name on this cover sheet and the four question sheets. As you turn through the pages, look for the easy questions — do them first. Remember that this exam is only 50 minutes long!

- You need not simplify your answers unless you are specifically asked to do so.
- It is essential to write legibly and *show your work*.
- If your work is absent or illegible, and your answer is not perfectly correct, then no partial credit can be awarded.
- Completely correct answers which are given without justification may receive little or no credit.

During this exam, you are not allowed to use calculators or consult your notes or books.

Problem	Maximum	Your Score
1	11	
2	12	
3	13	
4	9	
Total	**45**	

At the conclusion of the exam, hand in this exam paper to your TA.

Your Name: ______________________________

1a *(5 points)*. Write the repeating decimal $0.\overline{461538}$ as a quotient of two integers.

1b *(6 points)*. Evaluate $\displaystyle\int_{-1}^{1} \frac{dx}{|x|^{5/6}}$.

Your Name: ______________________________

2a *(5 points)***.** Find $\displaystyle\int \frac{dx}{x^{1/2} + x^{3/2}}$.

2b *(7 points)***.** Compute $\displaystyle\int_0^\infty x^2 e^{-x}\,dx$.

Your Name: ______________________________

3a *(6 points)***.** Find any two of the constants A, B, C, D:

$$\frac{x^3 + 2x^2 - 3x - 5}{(x^2 + 3x + 2)^2} = \frac{A}{(x+1)^2} + \frac{B}{x+1} + \frac{C}{(x+2)^2} + \frac{D}{x+2}.$$

3b *(7 points)***.** The curve $y = \sqrt{x}$ $(0 \le x \le 1)$ is rotated about the x-axis. Find the area of the resulting surface.

Your Name: ______________________________

4 *(9 points)***.** Let $c_1 = 1$, and define $c_{n+1} = 1 + \dfrac{1}{c_n}$ for all $n \geq 1$. Thus $c_2 = 2$, $c_3 = \dfrac{3}{2}$, $c_4 = \dfrac{5}{3}$, ...

a. Is the sequence $\{c_n\}$ bounded? Is it monotonic? Explain.

b. It's a fact that there is a number $L \sim 1.61803$ such that $c_n \to L$ as $n \to \infty$. Using the equation $c_{n+1} = 1 + \dfrac{1}{c_n}$, show that $L^2 - L - 1 = 0$.