



MATH 113 – EXAM 2

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Time limit: 80 minutes

Name:

SID:

You may not consult any books or papers. You may not use a calculator or any other computing or graphing device other than your own head!

Unless instructed otherwise, you are required to justify all of your answers. An answer with no justification will receive little credit. Please write all answers in complete English sentences.

There are two extra blank pages at the end of the exam. You may use these for computations, but **I will not read them**. Please transfer all final answers to the page on which the question is posed.

GOOD LUCK!

Problem:	Your score:	Total points
1		10 points
2		25 points
3		18 points
4		25 points
5		22 points
Total:		100 points

1. (10 points) Warm-up.

(a) (5 points) Find the order of $\sigma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 6 & 1 & 5 & 3 & 7 \end{bmatrix} \in S_7$.

(b) (5 points) Give two examples of nontrivial proper subgroups of the dihedral group D_6 . (You do not need to prove that they are subgroups, but you do need to explain any notation you use.)

2. **(25 points) True/False.** Please determine whether the following statements are true or false. Please justify your answer: give a proof or counterexample.

(a) **(5 points)** If G is an abelian group, and $H \trianglelefteq G$ a normal subgroup, then G/H is an abelian group.

(b) **(5 points)** If G is a group, $Z(G)$ its center, and $G/Z(G)$ is cyclic, then G is abelian.

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(c) (5 points) The groups D_4 and $\mathbb{Z}_4 \times \mathbb{Z}_2$ are isomorphic.

(d) (5 points) If G and H are groups, then $Z(G \times H) = Z(G) \times Z(H)$.

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(e) (5 points) The set X of all odd permutations in S_n is a subgroup of S_n .

3. **(18 points)** Construct a nontrivial homomorphism as indicated in each of the following cases, if possible. If it is possible, please also compute the kernel. If it is not possible, please say why it is not possible.

(a) **(6 points)** $\phi_1 : D_4 \rightarrow S_5$.

(b) **(6 points)** $\phi_2 : S_3 \rightarrow \mathbb{Z}_3$.

(c) **(6 points)** $\phi_3 : \mathbb{Z}_{10} \rightarrow \mathbb{Z}_8$.

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4. (25 points) Let $G = GL_2(\mathbb{R})$, and consider the subgroup

$$H = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \right\}$$

of upper triangular matrices.

(a) (10 points) Show that the subset $K \subseteq H$ with $a = c = 1$ is a normal subgroup of H .

(b) (15 points) Determine what the quotient H/K is.

5. (22 points) Recall that if $H \leq G$ is a subgroup, then the normalizer of H is the set

$$\text{Nor}_G(H) = \{g \in G \mid gHg^{-1} = H\}.$$

(a) (7 points) Prove that $\text{Nor}_G(H)$ is a subgroup of G .

(b) (7 points) Prove that H is a normal subgroup of $\text{Nor}_G(H)$.

(c) (8 points) Let $G = D_4 = \{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$, and let $H = \{e, b\}$. Please compute $\text{Nor}_G(H)$.