MATH 185 Spring 2001 Prof. Croot

Midterm 1

1.

a. Show that

$$\sum_{j=0}^{n-1}(w^a)^j = \begin{cases} n, & \text{if } a = 0; \\ 0, & \text{if } 1 \le a \le n-1. \end{cases}$$

where $w = e^{2\pi i/n}$, $n \ge 2$.

b. Suppose $f(z) = a_0 + a_1 z + \cdot + a_4 z^4$. Show that if

$$|f(w^j)| \le A, \ j = 0, 1, ..., 4,$$

where $w = e^{2\pi i/5}$, then $|a_0| \le A$.

2. Use the $\epsilon - \delta$ limit definition to prove that

$$\lim_{z \to i} z^2 + 1 = 0.$$

3.

a. Show that $f(z) = |x|^2 - iy^2$, $z = x + iy$, is not entire.

(Part b was omitted.)

c. Find a harmonic conjugate of $u(x, y) = x^3 - 3xy^2 + 2x^2 - 2y^2 + 3$.

d. Prove that $\cos(\sin(\overline{z})) = \overline{\cos(\sin(z))}$, for all $z \in \mathbf{C}$.

4.

a. Find the values of $\sin^{-1}(-1 + i\sqrt{3})$ (express in terms of $\ln(x)$).

b. State where

$$f(z) = \frac{\text{Log}(z+1)}{z+1}$$

is analytic

c. Show that $z^a z^b = z^{a+b}$ (for any fixed branch of $\log z$).

5. Prove

$$\left| \int_0^1 \sin(\alpha + it^2) dt \right| \le e - 1,$$

for any real α.

Bonus: Let $f(z) = a_0 + \cdots + a_{n-1} z^{n-1}$. Show

$$\sum_{j=0}^{n-1} |f(e^{2\pi ij/n})| \le n \sqrt{\sum_{j=0}^{n-1} |a_j|^2}.$$