

NAME _____

TA's name or section number _____

MATH 1B Second Midterm Fall 2001

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There are 200 points altogether. The first 8 questions are multiple choice, each worth 12 points. Choose the most correct answer to each question and mark the corresponding box in the grid below. Mark only one box per question. No partial credit.

Question	a	b	c	d	e
1					
2					
3					
4					
5					
6					
7					
8					

	MC		_____
	1		_____
TA use only:	2		_____
	3		_____
	Total		_____

Multiple choice questions:

1) Which of the following is correct for any convergent series $\sum_{n=1}^{\infty} a_n$ with positive terms?

- (a) $(\sum_{n=1}^{\infty} a_n)^2 = \sum_{n=1}^{\infty} a_n^2$
- (b) $\sum_{n=1}^{\infty} (a_n)^{1/n}$ converges.
- (c) $\sum_{n=1}^{\infty} 2^{-n} a_n$ converges.
- (d) $\sum_{n=1}^{\infty} 2^n a_n$ converges.
- (e) $\sum_{n=1}^{\infty} x^n a_n$ converges for all x .

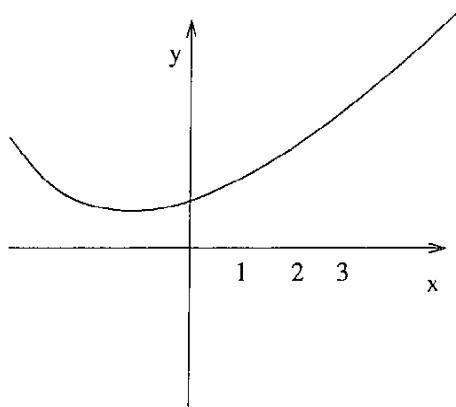
2) Which of the following is correct?

- (a) If $\sum_{n=1}^{\infty} a_n$ converges, the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$ is at least 1.
- (b) No power series $\sum_{n=1}^{\infty} a_n x^n$ can converge absolutely for all values of x .
- (c) The ratio test will always decide if a series converges or not.
- (d) The n th root test will always decide if a series converges or not.
- (e) Rugby is the most popular sport in the United States.

3) Which of the following is correct?

- (a) $y = 2e^x + C$ is the general solution of the differential equation $\frac{dy}{dx} = 2y$.
- (b) The differential equation $(\frac{dy}{dx})^2 = x^2 - 2$ has solutions for all x .
- (c) A separable equation $\frac{dy}{dx} = g(x)f(y)$ can always be solved in terms of elementary functions if $g(x)$ and $f(y)$ are elementary functions of x and y .
- (d) $\frac{dy}{dx} = e^{x+2y}$ is a separable differential equation.
- (e) $\frac{dy}{dx} = x + 2y$ is a separable differential equation.

4) The following graph could represent a solution of which differential equation?



- (a) $\frac{d^2y}{dx^2} = 4 - (y + x)^2$
 (b) $\frac{dy}{dx} = e^{xy}$
 (c) $\frac{d^2y}{dx^2} = x$
 (d) $\frac{d^2y}{dx^2} = 4 + (y + x)^2$
 (e) $\frac{d^2y}{dx^2} + e^y = 0$

5) If a_n is a sequence of positive numbers such that $L = \lim_{n \rightarrow \infty} (a_n/a_{n+1})$ and $M = \lim_{n \rightarrow \infty} (a_n)^{1/n}$ both exist and are nonzero then:

- (a) $L = M$ since they are both the inverse of the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$
 (b) $L = M^{-1}$ since they are both equal to the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$
 (c) $L^{-1} = M^{-1}$ since they are both the inverse of the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$
 (d) L/M is the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$
 (e) M/L is the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$

6) The 6th derivative of $(x - 2)^2 \ln(3 - x)$ at $x = 2$ is

- (a) 720
- (b) -720
- (c) $64/720$
- (d) $-64/720$
- (e) -180

7) A tank contains 30kg of salt dissolved in 4000 L of water. Brine that contains 0.02 kg of salt per liter of water enters the tank at a rate of 50 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate. If $S(t)$ is the amount of salt in kg in the tank at time t (in minutes), then

- (a) $\frac{dS}{dt} = \frac{150 - S(t)}{200}$
- (b) $\frac{dS}{dt} = 30 - \frac{S(t)}{60}$
- (c) $\frac{dS}{dt} = \frac{4000 - S(t)}{60}$
- (d) $\frac{dS}{dt} = \frac{1 - S(t)}{80}$
- (e) $\frac{dS}{dt} = 1 - \frac{S(t)}{80}$

8) Which curves are orthogonal trajectories for the family $y = \frac{k}{1+x^2}$?

- (a) $e^{y^2} = Cxe^{x^2/2}$
- (b) $y = x + C$
- (c) $y = x - C$
- (d) $y = x^2 + C$
- (e) $y = -x^2 + C$

The next three questions are *not* multiple choice. Show your reasoning and give your answers in the space provided.

NOTE THAT THERE ARE QUESTIONS ON BOTH SIDES OF THE PAGE!!!!

1. (30 points) Evaluate the following indefinite integral as a power series and give its radius of convergence:

$$\int \frac{\ln(1-x)}{2x} dx$$

2.(40 points) Find the interval of convergence, including determination of convergence at the end points, for the following power series:

$$\sum_{n=1}^{\infty} \frac{2^n (3x - 2)^n}{\sqrt{n+3}}$$

3.(34 points) Find the solution of the differential equation

$$\frac{dv}{dt} = 3t^2e^{t^2} + 2tv$$

which satisfies $v(0) = 5$.