Midterm #2

Math 121A (Section 2) - Fall 2001 M.Tokman

Each problem counts 20 points

Problem # 1. Given the equations

$$s + t = x,$$
$$s^2 + t^2 = y$$

compute $(\frac{\partial t}{\partial x})_y$ at the point $(s, t, x, y) = (1, -1, 0, 2)$.

Problem # 2. Let

$$f(z) = \frac{1}{(z+2)(z+1)^2} + e^{\frac{1}{z}},$$

(a) Identify all the singularities of $f(z)$ and specify the type of each of the singularities.

(b) Compute all possible Laurent series expansions of $f(z)$ around $z = 0$ and specify the region of convergence for each of the series. (*Hint:* The series $(1+w)^p = 1 + pw + \frac{p(p-1)}{2!}w^2 + \frac{p(p-1)(p-2)}{3!}w^3 + \ldots$, where $p = -1, -2, \ldots$, converges for $|w| < 1$).

(c) Evaluate residues of $f(z)$ at $z = 0$, $z = -1$ and $z = 3$.

Problem # 3. Evaluate the following integral using the residue theorem:

$$\int_0^\infty \frac{x^{1/3}}{x^2+1} dx.$$

Problem # 4. (a) Define what it means for a function $f(z) = u + iv$ to be analytic at a point $z = z_0$.

(b) Assume $f(z)$ is analytic at $z = z_0$. State and derive the *Cauchy-Riemann conditions.*

(c) Show that if the function $f(z) = u + iv$ is analytic in some region of a complex plane then its real and imaginary parts are harmonic functions in that region.

Problem # 5. The temperature T at each point (x, y) of a circular plate $x^2 + y^2 \leq 1$ is given by $T = 2x^2 - 3y^2$. Find the hottest and coldest points of the plate.