Fall 2001 Professor Borcherds 1st Midterm

Math 121A midterm, Thursday September 27, 9:40-11:00.

Please make sure that your name is on everything you hand in.
You are allowed calculators and 1 page of notes.
Answer as many questions as you can.
All questions have about the same number of marks.

1. Test the series $\sum_{n=1}^{\infty} n/(n^2+1)$, $\sum_{n=2}^{\infty} 1/n\log(n)$ and $\sum_{n=1}^{\infty} 100^n/n!$ for convergence.

2. Evaluate $\lim_{x\mapsto 0}(1-e^{x^3})/x^3$.

3. Express each of the following complex numbers in the form $a+ib$ for a and b real: $(1+i)/(2+i)$, $(1+i)^{10}$, $e^{3\pi i/2}$.

4. Evaluate $1+\cos(\theta)+\cos(2\theta)+\cdots+\cos(n\theta)$.

5. If $x^3+ay=b$ and $y^3+bx=a$ find $(\partial x/\partial a)_b$, $(\partial x/\partial a)_y$, $(\partial a/\partial x)_b$, $(\partial a/\partial x)_y$ at $(x,y,a,b)=(-1,2,3,5)$.

6. If $xe^y=ye^x$ find dy/dx and d^2y/dx^2 for $y\neq 1$.

7. Find all possible values of i^{1+i}.

8. Find a formula for $\cos(5\theta)$ as a polynomial in $\cos(\theta)$ and $\sin(\theta)$.