Final Exam

Math 121A (Section 2) - Fall 2001 M.Tokman

Each problem counts 10 points

Problem # 1. Solve the initial value problem using Laplace transform

$$y'' - 3y' + 2y = 10e^{5t}, \qquad y(0) = 1, \quad y'(0) = 5.$$

Problem # 2. The following periodic function $f(x)$ is defined over one period as $f(x) = 1 - x$ for $0 < x < 2$.

(a) Sketch several periods of $f(x)$ and expand it in an appropriate Fourier series.

(b) Write Parseval's relation for the Fourier series of $f(x)$.

Problem # 3. Find out in which quadrants the roots of the following equation lie

$$z^3 + z^2 + 4z + 9 = 0$$

Problem # 4. Show that if $f(x)$ is an odd function then its Fourier transform $g(\alpha)$ is also an odd function.

Problem # 5. Given

$$u = \int_x^{y-x} \sin(y-t)dt$$

Find $(\frac{\partial u}{\partial x})_y$, $(\frac{\partial u}{\partial y})_x$ and $(\frac{\partial y}{\partial x})_u$ at $x = \pi/2$, $y = \pi$.

Problem # 6. Given

$$f(x) = e^x + \ln(2x)$$

(a) Find Maclaurin expansion of $f(x)$.

(b) Find the interval of convergence of the Maclaurin series in (a) (including end points tests!). Justify your answer by mentioning the theorems/tests that you use to draw conclusions about convergence, and state explicitly if the convergence is absolute or conditional.

Problem # 7. Find the principal value of the integral

$$\int_0^\infty \frac{x \sin x}{9x^2 - \pi^2} dx.$$

Problem # 8. Sove the following boundary value problem using Green's function

$$y'' + 9y = \sin 2x, \qquad y(0) = 0, \quad y(\pi/2) = 0.$$

Problem # 9. Given

$$f(z) = \frac{z^{3/4}}{(z-1)^2(z^2+9)}.$$

(a) Specify under what conditions and where on a complex plane $f(z)$ is analytic.

(b) Identify all points where $f(z)$ is singular and specify the types of the singularities.

(c) Pick a branch of $f(z)$ and compute residues of this branch at $z = 1$, $z = 3$ and $z = 3i$.

Problem # 10. Let $u(x, t)$ satisfy the following equations

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \tag{1}$$

$$u(x, 0) = 0 \tag{2}$$

$$u(x, t) \to 0 \quad \text{as} \quad x \to \pm\infty. \tag{3}$$

(a) Laplace transform the equation (1) and write the boundary conditions satisfied by the Laplace transform of $u(x, t)$.

(b) Fourier transform the equation (1) and write the initial conditions satisfied by the Fourier transform of $u(x, t)$.